



## Robust iterative learning control via continuous sliding-mode technique with validation on an SRV02 rotary plant

Wen Chen<sup>a,\*</sup>, Yang-Quan Chen<sup>b</sup>, Chih-Ping Yeh<sup>a</sup>

<sup>a</sup>Division of Engineering Technology, Wayne State University, Detroit, MI 48202, USA

<sup>b</sup>Center for Self-Organizing and Intelligent Systems (CSOIS), Department of Electrical Engineering, Utah State University, 4120 Old Main Hill, Logan, UT 84322-4120, USA

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### ABSTRACT

This paper is to present a new design of robust Iterative Learning Control (ILC) for the purpose of output tracking using continuous sliding mode technique. The main feature of the design is that the controller signal is continuous due to the use of integral and employment of second-order sliding mode technique. The proposed ILC is more robust to noises and disturbances than the saturation approximation of the traditional sliding mode control because the control amount required to maintain the region of convergence is less. The robust ILC is suggested and the convergence of output-tracking error is also proven. The experimental results have clearly exhibited the excellent output-tracking performance by the continuous second-order sliding-mode-based robust iterative learning control.

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### 1. Introduction

Iterative Learning Control (ILC) is a control strategy for systems that execute the same trajectory, motion or operation repetitively. ILC attempts to improve the transient responses by adjusting control inputs during next system operation based on the errors observed in past operations.

Robustness, against uncertainties, time-varying, and/or stochastic noises and disturbances, includes problems such as disturbance rejection, and stochastic affects. Ref. [1] improves the ILC convergence speed for time-varying linear systems with unknown and bounded disturbances using the predicted errors. Time-periodic and non-structured disturbances are compensated for in [2] using a simple recursive technique. For general ideas about robust ILC, refer to Moon et al. [3] for the linear systems and see [4–8] for the nonlinear systems.

At present, the design of robust ILC often adopts standard  $H_\infty$  control techniques [9–11]; that is, given a system, an ILC controller gain is calculated using  $H_\infty$  synthesis. According to Norrlof and Gunnarsson [12], the main drawback of this approach is that the obtained ILC controllers are causal and the strength of ILC is related to the non-causality of its controllers. In recent years, more robust ILC schemes have been addressed. Research papers related to this topic can be found in [13–20], just to name a few.

Particularly, a robust ILC synthesizing learning control and sliding mode technique with the help of Lyapunov direct method is

proposed in [7]. The learning control is applied to the structured uncertainties while the variable structure scheme is to handle the unknown and unstructured uncertainties to ensure the global asymptotic stability. Another similar work is suggested in [21], where a Learning Variable Structure Control (LVSC) is formalized by combining variable structure control, as the robust part, and learning control, as the intelligent part. The proposed LVSC system achieves both uniform convergence of the tracking-error sequences to zero and that of the learning control sequences to the equivalent control.

In the aforementioned two research papers, saturation functions are employed to avoid discontinuity and eliminate the undesired chattering caused by the traditional Sliding Mode Control (SMC). This is because the discontinuous control signal will damage actuators or control devices in practice. The problem is that once the error signals exceed the designated boundary layer, a signum function is in charge of the control action. Hence, the saturation function itself can reduce the chattering to an extent that when the tracking-error signal is within the boundary. Therefore, the saturation function cannot avoid the discontinuity completely.

Higher-order sliding mode technique is able to eliminate the discontinuity with enhanced accuracy and robustness to disturbances [22–26]. In other words, compared with the saturation approximation of the traditional SMC, second-order sliding mode control is continuous and requires less amount of control efforts to maintain the operation within the region of convergence due to noises and disturbances.

This paper is to present and validate a robust ILC using continuous second-order sliding mode technique so that control signals

\* Corresponding author.

E-mail address: [wchen@wayne.edu](mailto:wchen@wayne.edu) (W. Chen).

are continuous and therefore chattering is reduced; thus, the continuous robust ILC can be applied broadly without damaging actuation devices.

This paper is organized as follows: in Section 2, the considered nonlinear system is illustrated and the objective of this paper is also addressed. The sliding surface and the controller design are described in Section 3. The convergence of the output-tracking error is also proven using Lyapunov direct method in the same section. An experiment is included to demonstrate the effectiveness of the proposed robust ILC. At last, concluding remarks are made in Section 5.

## 2. System formulation

Consider the following higher-order single-input and single-output nonlinear dynamical system described by

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t), \quad i = 1, \dots, m-1, \\ \dot{x}_m(t) &= \theta^T(t)\zeta(x, t) + b(x, t)u(t) + d(t), \end{aligned} \quad (1)$$

where the measurable system state  $x(t) = [x_1, x_2, \dots, x_m]^T$ ,  $u(t)$  and  $y(t)$  are the control input and system output, respectively,  $b(x, t)$  is a known non-zero function,  $\theta(t)$  is a  $p \times 1$  unknown and time-varying function to be learnt,  $\zeta(x, t)$  is a known vector-valued function with dimension of  $p \times 1$ . The variable  $d(t)$  represents the unknown disturbance.

**Assumption 1.** The desired output trajectory  $y_d(t)$  is differentiable with respect to time  $t$  up to the  $m$ th order on a finite time interval  $[0, T]$ , and all of the higher-order derivatives are available.

**Assumption 2.** The unknown disturbance variable  $d(t)$  is bounded such that

$$|d(t)| \leq b_d, \quad \forall t \in [0, T],$$

where  $b_d$  is a known positive constant.

**Assumption 3.** The initial condition  $e(0) = \dot{e}(0) = \ddot{e}(0) = \dots = e^{(m)}(0) = 0$  at any iteration  $\forall t \in [0, T]$ , such that the sliding surface  $\sigma(0) = 0$ , where  $e(t)$  is the output tracking error that is defined as  $e(t) = y_d(t) - y(t)$ .

The control objective is to design a continuous second-order sliding-mode iterative learning controller,  $u(t)$ , for the uncertain nonlinear system (1) such that system output can follow a desired one with a prescribed accuracy  $\epsilon$  as follows:

$$\forall t \in [0, T], \quad |y_d(t) - y(t)| \leq \epsilon.$$

## 3. Main results

The underlying robust ILC is to learn and approach the unknown and time-varying function and leave the remaining unknown function to the robust control. The global asymptotic convergence with respect to iterations is established by Lyapunov direct method.

### 3.1. Derivation of sliding surface

For the considered system (1), a sliding surface dynamics is defined as follows:

$$\sigma(t) = c_1 e(t) + c_2 \dot{e}(t) + \dots + c_m e^{(m-1)} = \sum_{i=1}^m c_i e^{(i-1)}, \quad (2)$$

where  $c_m = 1, c_i, s(i = 1, \dots, m-1)$  are coefficients of a Hurwitz polynomial, and  $e(t) = y_d(t) - y(t) = y_d(t) - x_1(t)$ .

Taking derivatives with respect to time  $t$  on both sides of (2), it is obtained:

$$\dot{\sigma}(t) = c_1 \dot{e}(t) + c_2 \ddot{e}(t) + \dots + c_m e^{(m)} = \sum_{i=1}^m c_i e^{(i)}. \quad (3)$$

Considering the fact that  $e(t) = y_d(t) - x_1(t)$ , the above equation can be further expanded:

$$\begin{aligned} \dot{\sigma}(t) &= c_1 [\dot{y}_d(t) - \dot{x}_2(t)] + c_2 [\ddot{y}_d(t) - \ddot{x}_3(t)] + \dots \\ &\quad + \left[ y_d^{(m)}(t) - \theta^T(t)\zeta(x, t) - b(x, t)u(t) - d(t) \right] \\ &= \sum_{i=1}^m c_i y_d^{(i)} - \sum_{i=1}^{m-1} c_i x_{i+1} - \theta^T(t)\zeta(x, t) - b(x, t)u(t) - d(t). \end{aligned} \quad (4)$$

The above equation can be further interpreted as the sliding variable dynamics. The condition,  $\sigma(t) = 0$ , defines the system motion on the sliding surface. The control signal,  $u(t)$ , is to be designed as an iterative and continuous control input signal. The task of this work is to design such a continuous and iterative control input to steer the sliding surface to be convergent to a region in finite time interval.

### 3.2. Design of the robust ILC using integral and continuous sliding mode technique

In Levant [24] and Levant [23], the second-order sliding-mode concept is originated. It is further developed in [25]. In reference to these work, an ILC via continuous second-order sliding-mode concept, at  $k$ th iteration, is designed as follows:

$$u_k(t) = b^{-1}(x_k, t) \left( \sum_{i=1}^m c_i y_d^{(i)}(t) - \sum_{i=1}^{m-1} c_i x_{i+1,k} - \hat{\theta}_k^T(t)\zeta(x_k, t) - v_k(t) + \alpha_1 |\sigma_k|^{1/3} \text{sgn}(\sigma_k) + \alpha_3 \sigma_k(t) \right), \quad (5)$$

where  $k$  indicates the number of iterations,  $x_k(t) = [x_{1,k}, x_{2,k}, \dots, x_{m,k}]^T$ ,  $\alpha_1, \alpha_2$ , and  $\alpha_3$  are positive constants,  $|\cdot|$  is the absolute value,  $\text{sgn}$  is the signum function,  $\hat{\theta}(t)$  is the recursive control part that is used to learn the unknown function  $\theta(t)$  and generated by the following update law

$$\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) - q \zeta(x_k, t) \left( \frac{4\eta}{3} |\sigma_k|^{1/3} \text{sgn}(\sigma_k) + \gamma \sigma_k(t) \right), \quad (6)$$

where  $q, \eta$  and  $\gamma$  are positive constants.

The variable  $v(t)$  is an integral term that is defined below:

$$\dot{v}_k(t) = -\beta_1 \sigma_k(t) - \beta_2 |\sigma_k|^{1/3} \text{sgn}(\sigma_k), \quad (7)$$

where  $\beta_1$  and  $\beta_2$  are positive constants.

**Remark 1.** Controller (5), together with (6) and (7), defines the continuous second-order sliding-mode ILC because  $|\sigma_k|^{s/3} \text{sgn}(\sigma_k)$ , where  $s = 1, 2$ , are two continuous functions. Moreover,  $|\sigma_k|^{1/3} \text{sgn}(\sigma_k)$  is integrated in (7) such that  $v(t)$  is absolutely a continuous function. In summary, controller (5) is a continuous signal; it therefore leads to a chattering-free control action.

Therefore, the sliding surface dynamics (4) can be simplified by inserting the ILC law (5):

$$\begin{aligned} \dot{\sigma}_k(t) &= -\alpha_3 \sigma_k(t) + \Phi_k^T(t)\zeta(x_k, t) + v_k(t) - d_k(t) \\ &\quad - \alpha_1 |\sigma_k|^{2/3} \text{sgn}(\sigma_k), \end{aligned} \quad (8)$$

where  $\Phi_k(t) = \hat{\theta}_k(t) - \theta(t)$ .

The sliding surface dynamics (8) implies that the integral term  $v(t)$  is used to attenuate the effect of the unknown disturbance  $d(t)$ .

**Remark 2.** According to Brown et al. [26], the second-order SMC is more robust to noises and disturbances than the saturation approximation of the traditional SMC because the control amount required to maintain the region of convergence is less.

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