# Performance analysis for closed-loop production systems with unreliable machines and random processing times

C.H. Paik[*], H.G. Kim, H.S. Cho

*Division of Mechanical and Industrial System Engineering, Dongeui University, Pusan 614-714, South Korea*

## Abstract

In this study, we propose some simple but effective throughput-approximation methods for finite-buffered closed-loop production systems with unreliable machines and exponentially distributed processing times. The proposed approximation methods are based on decomposition and aggregation principles. According to the method representing the decomposed systems, three different approximation methods are developed, along with a simple upper bounding method for throughputs. Extensive computational experiments are performed to show the effectiveness of the approximation and upper bounding methods. © 2002 Elsevier Science Ltd. All rights reserved.

*Keywords*: Closed-loop queuing networks; Approximation; Decomposition; Throughput

## 1. Introduction

Finite-buffered closed-loop queuing networks with machine failure (FCQNs-F) have been major tools for evaluating the performances of production systems (Frein, Comnault, & Dallery, 1996; Gershwin & Berman, 1981). Two major characteristics of FCQNs-F are the blocking and machine failure phenomenon. The queuing networks with this type of characteristics are basically unsolved except for some special cases (Dallery & Frein, 1993; Frein et al., 1996). Therefore, exact solutions can be obtained by means of numerical techniques requiring time and space consumption, and the numerical technique can be used only for small networks. As a result, a considerable amount of effort has been devoted to the development of approximation methods (Choong & Gershwin, 1987; Dallery & Frein, 1989, 1993).

Only a few studies have been dedicated to the analysis of FCQNs-F. But even these studies are confined to the FCQNs-F with deterministic processing times (Frein et al., 1996) or the open-loop queuing networks (Gershwin & Berman, 1981). To our knowledge, there is no study dealing with FCQNs-F with random processing times. In this study, we propose some simple but effective

* Corresponding author.
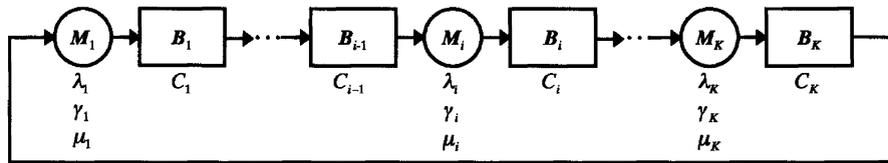 *E-mail address:* chpaik@hyomin.dongeui.ac.kr (C.H. Paik).

Fig. 1. An FCQN-F model.

throughput-approximation methods for FCQNs-F with random (specifically, exponential) processing times, along with an upper bounding method for the throughput.

This paper is organized as follows. The FCQNs-F model is introduced in Section 2. Section 3 contains a detailed description of the approximation methods, and Section 4 presents a simple through-put bounding method. Some numerical results are discussed in Section 5. Conclusions are drawn in Section 6.

## 2. Model description

Consider an FCQN-F consisting of a series of $K$ servers separated by $K$ intermediate finite buffers, as shown in Fig. 1. Customers (parts) visit successively server $M_1$, buffer $B_1$, server $M_2$, buffer $B_2$,..., server $M_K$, buffer $B_K$, server $M_1$, and so on. Let $N$ denote the number of customers circulating in the FCQN-F. The service time at server $i$ ($i = 1, 2, ..., K$) follows an exponential distribution with rate $\mu_i$. It is assumed that each server can fail only while giving a service to a customer and the repair of the server would be initiated at the instant of its failure. The times to failure and to repair are exponentially distributed with rates $\lambda_i$ and $\gamma_i$, respectively. Throughout this study, we use the following convention for the indexes of servers and buffers: $i - 1$ is $K$ if $i = 1$ and $i + 1$ is 1 if $i = K$.

Due to the finiteness of buffers, a server may be blocked if upon service completion of a customer the downstream buffer is full (blocking after service). The capacity of buffer $B_i$ which includes the working space of downstream server $M_{i+1}$, is denoted by $C_i$. To avoid some trivial cases, we consider only the FCQNs-F with populations $N$ ranging from ($C_{min} + 1$), where $C_{min}$ is the smallest buffer capacity of FCQN-F, to ($C - 1$) ($C$ is the sum of all buffer capacities).

## 3. Approximation methods

### 3.1. Decomposition method

The principle of the decomposition method is to decompose the original system $L$ into a set of $K$ subsystems $L(i)$ ($i = 1, 2, ..., K$). Each subsystem $L(i)$ consists of an upstream server $M_u(i)$, a downstream server $M_d(i)$, and an intermediate buffer $B(i)$, as shown in Fig. 2. From now on, integer index will be used to represent parameters of the original system $L$, while integer in parenthesis will represent the parameters of subsystem $L(i)$. Servers $M_u(i)$ and $M_d(i)$, respectively, represent the behaviors of the upstream and downstream parts of buffer $B_i$ in the original system $L$. In $L(i)$, the upstream server $M_u(i)$ is never starved and the downstream server $M_d(i)$ is never blocked. Let $\lambda_u(i)$, $\gamma_u(i)$, and $\mu_u(i)$