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Mobility analysis of a complex structured ball based on mechanism decomposition and equivalent screw system analysis

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Abstract

The complex and articulated ball is one of the popular artifacts for collections and is highly expandable and collapsible. This paper investigates the mechanism structure of this magic ball by hypothetically decomposing the mechanism into kinematic loops and chains and subsequently into basic kinematic sub-chains, leading to the analysis of the mobility of the ball mechanism. Dismantling those kinematic chains which do not contribute to the mechanism mobility, the magic-ball mechanism is decomposed into a number of kinematic chains, and further disintegrated into two distinct types of elementary platforms with three and four legs respectively. The theory of the relationship of a screw system and its reciprocal system is then used to identify both common constraint and virtual constraint in each of the elementary platforms and to examine the mobility of those platforms. The analysis is then extended to the mobility analysis of the closed-loop circular kinematic chain and supplementary chains. A systematic analysis is hence produced in mechanism decomposition and in the analysis of virtual constraints. The paper produces a theoretical basis for the mobility analysis of the mechanism using mechanism decomposition and screw system analysis.

Keywords: Mobility; Screw system; Constraint; Mechanism; Kinematic chain; Platform; Parallel mechanism

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1. Introduction

Mobility is the total degrees of freedom which need to be controlled in a mechanism for every link to be in a specific position and is related to the connectivity which is defined as the number of degrees of freedom between two given links of a mechanism, provided that the mechanism is constructed accurately that the expected motion can be achieved. The earliest analytical finding in the mobility is attributed to Grübler and Kutzbach [1]. A generic formula used to describe the mobility of a mechanism was proposed

$$m = b(n - g - 1) + \sum_{i=1}^{g} f_i, \tag{1}$$

where n is the number of links, g the number of joints, f_i the degree of freedom of the ith joint, and b the mobility coefficient. The mobility coefficient b has two numbers where 3 is for a planar mechanism and 6 for a spatial mechanism.

The complexity of mechanisms incurs difficulties in mobility analysis. To this extent, Shoham and Roth [2] decomposed a mechanism into a single simple closed loop and a group of in-parallel serial chains. Thus, loop analysis can be taken into account in the mobility calculation as

$$m = \sum_{i=1}^{g} f_i - bl, \tag{2}$$

where l is the total number of independent loops and the mobility coefficient b can be taken as the order of every independent loop. This coefficient or loop order b was related by Waldron [3] in Eq. (1) to the order of a screw system formed by joint axes of a mechanism. Hence, in the above two equations, coefficient b can be extended to taking any integer between 3 and 6.

This led to taking account of the special geometric arrangement of a mechanism in the study of its mobility and to the use of screw system geometry [4–6]. A typical case was demonstrated by Hunt [7] to use the linear complex to determine instantaneous screw axes in spatial mechanisms and demonstrate how the existence of many over-constrained linkages can be explained.

The screw system analysis [5,6] helps mobility analysis in spatial mechanisms [8] and in metamorphic mechanisms [9]. In the study of a parallel mechanism, the mechanism can be decomposed into several legs which comprise kinematic chains. Lines of joint axes of a kinematic chain forms a screw system of motion of this kinematic chain. The reciprocal system of this screw system gives the constraint wrenches of the kinematic chain. The aggregate of the screw system of motion of all kinematic chains gives the screw system of the mechanism. The aggregate of the reciprocal systems [10] of kinematic chains gives constraint wrenches which limit the degree of freedom of the mechanism. In the latter set of the aggregate, the constraint wrench which is shared by every kinematic chain is the common constraint [11,12]. The common constraint is reciprocal to the screw system of the mechanism. The order of the screw system of the mechanism is then reduced by the number of the common constraints from a total number of 6 [13]. The remaining constraint wrenches of the aggregate form a complementary constraint system. Any constraint wrench from a kinematic chain which is dependent on the complementary constraint system is a virtual constraint [14] which is a redundant constraint and the removal of which does not affect the motion of the mechanism.