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Minimax distribution free procedure with backorder price discount

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Abstract

The inventory models analyzed in this paper explore the problem in which the lead time and ordering cost reductions are inter-dependent in the continuous review inventory model with backorder price discount. The objective is to minimize total related cost by simultaneously optimizing the order quantity, reorder point, lead time and backorder price discount. Moreover, we assume that the mean and variance of the lead time demand are known, but its probability distribution is unknown. We apply a minimax distribution free procedure to find the optimal solution, and three numerical examples are given to illustrate the results.

Keywords: Backorder price discount; Lead time; Minimax distribution free procedure

1. Introduction

In classical economic order quantity (EOQ) model dealing with inventory problems, either using deterministic or probabilistic models, lead time is viewed as a prescribed constant or a stochastic variable. Therefore, lead time is not subject to control (see, e.g., Naddor, 1966; Johnson and Montgomery, 1974; Silver and Peterson, 1985). However, this may not be realistic. In many practical situations, lead time can be shortened at an added crashing cost; in other words, it is controllable. By shortening the lead time, we can lower the safety stock, reduce the stockout loss and improve the service level to the customer so as to increase the competitive edge in business.

Recently, several authors have presented various inventory models with lead time reduction. Initially, Liao and Shyu (1991) presented an inventory model in which the lead time is a unique decision variable and the order quantity is predetermined. Ben-Daya and Raouf (1994) extended Liao and Shyu's (1991) model by allowing both the lead time and the order quantity as decision variables. Ouyang et al. (1996) generalized Ben-Daya and Raouf's (1994) model and considered shortages with partial backorders, while Pan and Hsiao (2001) revised Ouyang et al.'s (1996) model to consider the backorder price discount as one of the decision variables.

It is noticed that the above papers Liao and Shyu (1991), Ben-Daya and Raouf (1994), Ouyang et al. (1996), Pan and Hsiao (2001) are all focused on the continuous review inventory model to derive the benefits from lead time reduction, and the ordering cost is treated as a fixed constant. In a recent paper,

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Ouyang et al. (1999) proposed two continuous review inventory models to study the effects of lead time and ordering cost reductions. We note that the lead time and ordering cost reductions in Ouyang et al. (1999) are assumed to act independently. However, this is only one of the possible cases. In practice, the lead time and ordering cost reductions may be related closely; the reduction of lead time may accompany the reduction of ordering cost, and vice versa. For example, according to Silver and Peterson (1985, p. 150), the implementation of electronic data interchange (EDI) may reduce the lead time and ordering cost simultaneously. Therefore, it is more reasonable to assume that the ordering cost reduction vary according to different lead times.

In the real market, as unsatisfied demands occur, we can often observe that some customers may prefer their demands to be backordered, and some may refuse the backorder case. There is a potential factor that may motivate the customers' desire for backorders. The factor is an offering of a backorder price discount from a supplier Pan and Hsiao (2001). In general, provided that a supplier could offer a backorder price discount on the stockout item by negotiation to secure more backorders, it may make the customers more willing to wait for the desired items. In other words, the bigger the backorder price discount, the bigger the advantage to the customers, and hence, a larger number of backorder ratio may result. This phenomenon reveals that, as unsatisfied demands occur during the stockout period, how to find an optimal backorder ratio through controlling a backorder price discount from a supplier to minimize the relevant inventory total cost is a decision-making problem worth discussing.

In this paper, we attempt to modify Pan and Hsiao's (2001) model for a minimax distribution free inventory model that includes a controllable backorder price discount and the reduction of lead time accompanies a decrease of ordering cost. For this case, we solve the problem by using the minimax distribution free approach, which was originally proposed by Scarf (1958). Recently, Gallego and Moon (1993) presented a new and very compact proof of the optimality of Scarf's (1958) ordering rule. Also, Hariga and Ben-Daya (1999), Moon and Choi (1995, 1997), Moon and Silver (2000), Ouyang and Wu (1998), Ouyang and Chang (2002), Ouyang et al. (2004), Silver and Moon (2001) applied this approach to some production/inventory models. Moreover, note that the previous works on distribution free approach and partial lost sales (or backorders) are well documented in Silver et al. (1998). In this study, the objective is to minimize the total related cost by optimizing the order quantity, reorder point, backorder price discount and lead time, simultaneously. Furthermore, the effects of parameters are also included and three illustrative numerical examples are given.

2. Notation and assumptions

The mathematical models in this paper are developed on the basis of the following notation and assumptions.

Notation

D = average demand per year
A_0 = original ordering cost (before any investment is made)
A = ordering cost per order, $0<A\leqslant A_0$
h = inventory holding cost per unit per year
Q = order quantity (a decision variable)
r = reorder point (a decision variable)
β = fraction of the shortage that will be backordered, i.e., backorder ratio, $0\leqslant\beta<1$
β_0 = upper bound of the backorder ratio
π_x = backorder price discount offered by the supplier per unit (a decision variable)
π_0 = marginal profit (i.e., cost of lost demand) per unit
L = length of lead time (a decision variable)
X = lead time demand
$f_X(x)$ = the probability density function (p.d.f.) of X with finite mean DL and standard deviation $\sigma\sqrt{L}$, where σ denotes the standard deviation of the demand per unit time
$E(\cdot)$ = mathematical expectation
x^+ = maximum value of x and 0, i.e., $x^+ = \mathrm{Max}\{x,0\}$.

Assumptions

(1) The reorder point r = expected demand during lead time + safety stock (SS), and SS = $k\times$ (standard deviation of lead time demand), i.e., $r = DL + k\sigma\sqrt{L}$, where k is the safety factor.
(2) Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point r.