Computers & Industrial Engineering 66 (2013) 976–987

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A deteriorating multi-item inventory model with price discount and variable demands via fuzzy logic under resource constraints ☆

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ARTICLE INFO

Article history:
Received 31 August 2011
Received in revised form 14 July 2013
Accepted 21 August 2013
Available online 4 September 2013

Keywords:
Fuzzy logic
Genetic algorithm
Fuzzy inference
Price discount
Resource constraints
Chance constrained technique

ABSTRACT

An inventory model of deteriorating seasonal products with Maximum Retail Price (MRP) for a wholesaler having showrooms at different places under a single management system is considered under random business periods with fuzzy resource constraints. The wholesaler replenishes the products instantaneously and earns commissions on MRP which vary with the ordered quantities following All Unit Discount (AUD), Incremental Quantity Discount (IQD) or IQD in AUD policy. Demand at showrooms are imprecise and related to selling prices by 'verbal words' following fuzzy logic. The wholesaler shares a part of commission with customers. The business periods follows normal distribution and converted to deterministic ones through chance constraint technique. The fuzzy space and budget constraints and fuzzy relations are defuzzified using possibility measures, surprise function and Mumdani fuzzy inference technique. The model is formulated as profit maximization for the wholesaler and solved using a real coded Genetic Algorithm (GA) and illustrated through some numerical examples and some sensitivity analysis. A real-life problem of a developing country is presented, solved using the above mentioned procedures and an appropriate inventory policy is suggested.

1. Introduction

In the existing literature of inventory, most of the models are developed under infinite time horizon. As per Gurnani (1985), the life of a particular item is not infinite due to the change of design, technological development, variation of inventory costs, customers' changing taste, etc. and this is very much true for the seasonal products in developing countries where preserving facilities are not available in plenty. For these seasonal products, even though the planning horizon is assumed as finite, in every season it fluctuates depending on some extraneous factors such as climatic conditions. This time period may be assumed to be random with a probability distribution. In the literature Maiti, Maiti, and Maiti (2006) and Roy, Pal, and Maiti (2009) have solved some inventory problems with random planning horizon having exponential distribution. Also Moon and Lee (2000) have presented an EOQ model under inflation and discounting with a random product life cycle.

In an inventory system, deterioration is an usual phenomenon. Mandal and Phaujdar (1989) presented an inventory model with deteriorating items. Roy, Maiti, Kar, and Maiti (2009) have done a research work of deteriorating items with stock dependent demand over random planning horizon. Also Bhunia and Maiti (1997) and Mahapatra and Maiti (2006) presented some inventory models for deteriorating items with time dependent demand and imprecise production time respectively.

In the present competitive market, the demand depends on the stock directly and also inversely on the selling price. Recently Widyadana, Cardenas-Barron, and Wee (2011) presented a deteriorating inventory problem with constant demand via a simplified approach. Also Giri, Pal, Goswami, and Chaudhuri (1996), Mandal and Maiti (2000) and others considered the demand as an indexed stock (i.e. $D = dq^{\beta}$, d and β are constants) dependent. But there are few research works with fuzzy demand depending on stock and selling price following fuzzy inference. Recently, some inventory models with rework for the defective products (Jamal, Sarker, & Mondal, 2004; Cardenas-Barron, 2007, 2008, 2009a, 2009b; Sarker, Jamal, & Mondal, 2008; Cardenas-Barron, Trevino-Garza, & Wee, 2012) have been presented in the literature.

Human knowledge is often represented imprecisely, vaguely and approximately. In our real life, some vague terms in the form of 'words' such as high, medium, and low, are used. The target of fuzzy inference process is to form it into natural language expressions of the type,

IF premise (antecedent) THEN conclusion (consequent).

There are two types of fuzzy inference systems: Mamdani-type (Mamdani & Assilina, 1975) and Sugeno-type (Ban, Gao, Huang, & Yin, 2007). These two types differ in the way by which output is determined. Mamdani's effort was based on Bellman and Zadeh's

☆ This manuscript was processed by Area Editor Alexandre Dolgui.

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http://dx.doi.org/10.1016/j.cie.2013.08.018

(1970) paper developing fuzzy algorithms for complex systems and decision processes. The main difference between Mamdani and Sugeno is that Sugeno output membership functions are either linear or constant where the Mamdani output is a fuzzy set. Since performance or satisfaction level of a perfect order cannot be judged in terms of discrete values, a Mamdani-type inference system is selected here for evaluating and aggregating the fuzzy rules.

Among the recently used optimization techniques, Genetic Algorithm (GA) is the most popular one. Some advantages can be pointed out for acceptability of this method. (i) GAs work with a population of solutions instead of a single solution and for this it gives more globalized solution. (ii) GAs do not require any auxiliary information except the objective function values. Also there are some classical direct search methods which work under the assumption that the function to be optimized is unimodal. GAs do not impose any such restriction. (iii) GAs use probabilistic guide lines for search where in most of the classical methods, fixed transition rules are used to move from one solution to another. Some research works (Zydallis, David, Veldhuizen, & Lamont, 2001) using GA process are available in the literature.

Inspite of several devolopment in the area of supply-chain models, there are still some gaps in the literature.

(1) There are very few supply-chain models for deteriorating items with fuzzy inference expressed verbally using 'words'.
(2) Till now, none has used three types of price discount (AUD, IQD, AUD in IQD) in a supply-chain model connecting through fuzzy inferences and sharing the part of the commission with customers.
(3) No supply-chain is available with MRP and commission on this following fuzzy rules.
(4) Use of random planning horizon is very limited and none has used it in connection with fuzzy inferences.
(5) Ga is not yet devoloped connecting random planning horizon, fuzzy logic and price discount.
(6) For the first time, surprise function, possibility for resource constraints are used in a supply-chain model.

In this paper an inventory model for some seasonal products is presented with a wholesaler and its m showrooms under a random planning horizon. The wholesaler purchases a number of items from a set of predetermined suppliers and supplies the items to the showrooms for sale in certain number of cycles to achieve the maximum profit. The suppliers offer some ranges of commission on MRP to the wholesaler and it is presented in three ways, in the forms of AUD, IQD or IQD in AUD. The wholesaler shares a part of this commission with his/her customers. Demands of the items at the open market depends on the discount given on MRPs of the items by some defined verbal fuzzy rules. Items considered here deteriorate at some fixed rates. Moreover the wholesaler has resource constraints in purchasing and storing the items due to limited budget and storage capacity. The model is formulated with a fuzzy space constraint in the form of possibility (Liu & Liu, 2002) and a crisp budget constraint in the form of surprise function. In seasonal business, the business periods are uncertain in stochastic sense. So the time horizons considered in this model are random with normal distribution and are evaluated by chance-constraint method. The fuzzy relations are defuzzified following Mamdani technique. The profit function formulated with respect to the wholesaler is maximized using a real coded Genetic algorithm. The model is illustrated with numerical examples. Some raw data regarding the model parameters (such as demand, and selling price) in a developing country are collected and represented in the from of fuzzy number and an inventory policy is developed. The novelty of the paper is that for the first time, a real-life supply chain/inventory model has been formulated and solved with price discount under random planning horizon and fuzzy inference concerning price and demand taking some imprecise and crisp resource constraints into account.

The rest of the paper is organized as follows. Sections 2–5 give some basic ideas, definitions and theorems about fuzzy logic, chance constraint method, possibility and construction of fuzzy number respectively. Section 6 contains the assumptions and notations made for the proposed model. In Section 7, there are some fuzzy rules considered for the problem and other calculations of the model. Section 8 presents the GA process in favor of the proposed model. A numerical experiment is performed in Section 9 and a real life application is presented in Section 10. Discussion and conclusion are made in the Sections 11 and 12 respectively.

2. Fuzzy logic and fuzzy inference

2.1. Definitions

Fuzzy expression: An n-dimensional fuzzy expression function is a mapping from $[0,1]^n$ to $[0,1]$. i.e., f: $[0,1]^n \rightarrow [0,1]$.

Fuzzy logic: The fuzzy logic is a logic represented by the fuzzy expression (formula) which satisfies the followings.

(i) Truth values, 0 and 1, and variable x_i ($\in [0.1], i = 1, 2, \ldots, n$) are fuzzy expressions.
(ii) If f is a fuzzy expression, $\sim f$ is also a fuzzy expression.
(iii) If f and g are fuzzy expressions, $f \wedge g$ and $f \vee g$ are also fuzzy expressions.

where $\sim$(*negation*), $\wedge$(*conjunction*) *and* $\vee$(*disjuction*) are used as in the classical logic. Thus according to Lukasiewicz, for $a, b \in [0,1]$

Negation $\sim a = 1 - a$
Conjunction $a \wedge b = Min(a,b)$
Disjunction $a \vee b = Max(a,b)$

2.2. Over view of fuzzy inference process

The term "inference" refers to a process of obtaining new information by using existing knowledge and it is commonly referred to as IF-THEN rule-based form. It typically expresses an inference such that if we know a fact (premise, hypothesis, antecedent), then we can infer or derive another fact called a conclusion (consequent) i.e.

"If x is a Then y is b".

Different steps of fuzzy inference process are

Fuzzification of input value: When a value of premise is given as an input, it must correspond to some one or more linguistic fuzzy sets with some membership values (see Fig. 1).

Rule Strength Calculation: After the inputs are fuzzified, the degree to which each part of the antecedent is satisfied for each rule is known. The degree of a rule is the rule strength of the corresponding rule. If there are more than one antecedent then the rule strength is calculated by the standard min operator

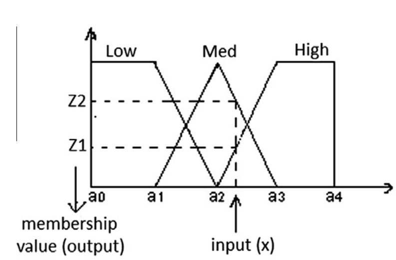

Fig. 1. Fuzzyfication of inputs.