ELSEVIER

Journal of Economic Dynamics & Control
26 (2002) 577–609

JOURNAL OF
Economic
Dynamics
& Control

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Approximating infinite-horizon models in a complementarity format: A primer in dynamic general equilibrium analysis☆

Morten I. Lau[a,b], Andreas Pahlke[c],
Thomas F. Rutherford[a,d,*]

[a]*The MobiDK Project, Danish Ministry of Trade and Industry, Copenhagen, Denmark*
[b]*Institute of Economics, University of Copenhagen, Denmark*
[c]*Institut für Energiewirtschaft und Rationelle Energieanwendung, University of Stuttgart, Germany*
[d]*Department of Economics, University of Colorado, CO 80309-0256, Boulder, USA*

Received 22 July 1998; accepted 12 July 2000

Abstract

We demonstrate the advantages of the complementarity formulation for approximating infinite-horizon equilibria in neoclassical growth models as compared with techniques originally developed for optimal planning models. The complementarity approach does not require an ex ante specification of the growth rate in the terminal period and is therefore suitable for models with endogenous growth or short time horizons. We also consider approximation issues in models with multiple infinitely lived agents. Changes in net indebtedness over a finite period are estimated as part of the model to obtain a precise approximation of the infinite-horizon equilibria with a small number of time periods. © 2002 Elsevier Science B.V. All rights reserved.

JEL classification: C68; D58; D91

Keywords: Intertemporal optimization; Infinite-horizon equilibria; Terminal constraints

☆We are grateful to Glenn W. Harrison, Tobias N. Rasmussen and an anonymous referee for helpful comments.

* Corresponding author. Tel.: + 1-303-492-5169; fax: + 1-303-492-8960.
E-mail address: rutherford@colorado.edu (T.F. Rutherford).

PII: S0165-1889(00)00071-3

1. Introduction

Approximation of infinite-horizon models has a long-standing tradition in the economics literature. Most of this literature deals with optimization methods. In this paper we demonstrate the usefulness of the complementarity format for approximating optimal saving and investment decisions in dynamic general equilibrium models. Our objective is pedagogic – the essential equations for a few models are presented in a compact and accessible format, along with computer programs which concretely illustrate the models. The approach is of interest to applied economists due to the availability of 'off the shelf' software for processing these models (see Rutherford, 1995, 1999a).

There are two key issues involved in approximating an infinite horizon equilibrium for a neoclassical growth model: (i) what is the size of the capital stock in the terminal period?, and (ii) who owns the terminal capital stock? We illustrate the advantages of the complementarity formulation for answering these questions as compared with techniques originally developed for optimal planning models.

We begin the paper with the classical Ramsey analysis of optimal economic growth under certainty. This is a natural starting point because of the generic representation of financial markets. The model represents a closed economy with perfect competition in all markets, a representative consumer, and a constant rate of technological progress. Although the model is well studied in the economics literature (see, for example, Blanchard and Fischer, 1989; Barro and Sala-i-Martin, 1995), analytical methods have limitations. Numerical methods are, of course, always required for empirical analysis of policy issues, and they can provide helpful insights into properties of alternative formulations.

In Section 2 we formulate the Ramsey model as a primal nonlinear program in quantities, as two different mixed complementarity problems (MCPs), and as a dual nonlinear program in prices. Preferences and technology are represented by utility and production functions in the primal formulation and by expenditure and cost functions in the dual model. The two MCP formulations can be interpreted as first-order necessary conditions for the nonlinear programming (NLP) models, and the complementarity problem associated with the dual nonlinear program is essentially Mathiesen's (1985) formulation of the Arrow–Debreu equilibrium model.

In Section 3 we consider methods of approximating the infinite horizon. We present a new method for approximating these equilibria in models with endogenous capital accumulation, and we demonstrate the advantages of this approach as compared with techniques based on optimization methods. The complementarity approach does not require an ex ante specification of the growth rate in the terminal period, and it is therefore suitable for models with endogenous growth or short time horizons. We illustrate in a few examples that