General equilibrium with endogenous uncertainty and default[☆]

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Abstract

We study the introduction of new assets that are defined in expected values rather than state by state. Individual default emerges naturally in an economy where such assets are introduced without completing all contingency markets. We further provide conditions under which individual default is propagated endogenously into a collective risk of widespread default in general equilibrium. We prove existence of a general equilibrium with endogenous uncertainty.
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1. Introduction

New financial instruments are introduced every day including indices, derivatives and innovative forms of government debt. They help manage risk and improve economic welfare. However, they can also increase macroeconomic volatility. The complexity of contractual obligation within a market can transmit individual risks and amplify them into correlated or collective risks. There are trade-offs arising from the gains and the losses created by financial innovation. This article

shows the connection between financial innovation and default, and it focuses on the propagation of default in complex markets.

Markets can magnify risk. As new assets are introduced, a creditor who is a victim of default in one transaction is unable to deliver in another, thereby causing default elsewhere. In this manner default by one individual leads, through a web of obligations, to a large number of defaults. Since new instruments create new webs of obligations, financial innovation is the precipitating factor. The transmission of default from one trader to another and from one market to another transmits individual risk and magnifies it into collective risk. Default by one individual leads to a collective risk of widespread default.

We introduce a formal framework based on *individual* and *collective* risk. We show how financial instruments that are introduced to manage *individual risk* often increase *collective risk*. The newly created uncertainty does not originate in nature, but from market forces. It is *endogenous uncertainty*, and is best formalized by a set of simultaneous decisions that affect market behavior as in *general equilibrium* analysis.[1]

Precisely how does financial innovation lead to collective default? We start from a large economy with an incomplete set of assets, where agents face individual risks. A new asset is introduced, whose payoffs are defined in terms of expected values rather than state by state. We call these 'statistical assets'; similar assets have been studied in Arrow and Lind (1970) and Malinvaud (1972, 1973), and we denote them Arrow–Lind–Malinvaud (ALM) assets. A typical example is provided by insurance contracts, which are valued based on their expected value. Such assets exist in large societies because of the inherent difficulties of dealing with contracts whose payoffs are contingent on each individual's state,[2] such as those in the Arrow–Debreu model. The next step is to show in Section 3 how individual default emerges with such ALM assets, and how individual default is propagated and magnified into a collective risk of widespread default once the new asset is introduced.

Since the value of a newly introduced ALM asset is determined in terms of statistics this creates states of default. For example, in Malinvaud (1972), the statistic is the expected number of people who are ill, and the random variable is the number of sick people. As the population size increases, the law of large numbers predicts that the random variable representing the number of sick people converges to a fixed proportion almost surely. Therefore in the limit, but only in the limit, insurance that is provided at actuarially fair prices – expected value – matches premium precisely to the insurance payments. However, when the economy is large but finite, no matter how close we are to the limit, the law of large numbers does not operate exactly. Therefore insurance contracts designed to deal with an exact proportion of sick people will not be able to cope with actual payments in those cases where the realized numbers exceed the limiting proportions. Insurance contracts offered at actuarially fair values (even with a premium) promise payments that exceed physical endowments, with small but positive probability. This is how default arises when ALM

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[1] The concept of *endogenous uncertainty* refers to uncertainty that depends on economic behavior along with nature's moves. Chichilnisky and Wu (1992) provided the first proof of existence of a market equilibrium with endogenous uncertainty, see also Chichilnisky and Heal (1993), Chichilnisky and Gruenwald (1995), Chichilnisky (1996) and Chichilnisky (1999). Kurz (1974) defined a research agenda of endogenous uncertainty. Recent studies on this topic include Svensson (1981), Chichilnisky, Dutta and Heal (1991), Chichilnisky (1996, 1999), Huang and Wu (1999), Kurz (1994), Kurz and Wu (1996), and Wu and Guo (2003, 2004).

[2] Cass et al. (1996) demonstrated that an appropriate combination of Arrow securities and mutual insurance policies can achieve efficient allocation in a world of individual and collective risks. However, in the real world there may not exist a complete set of such assets. Once we depart from the complete market economy of Cass, Chichilnisky and Wu (1996), default will occur.