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Journal of Banking & Finance 31 (2007) 2517–2534
Journal of BANKING & FINANCE
www.elsevier.com/locate/jbf

Coherent measures of risk from a general equilibrium perspective ☆

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Received 24 April 2006; accepted 24 October 2006
Available online 24 January 2007

Abstract

Coherent measures of risk defined by the axioms of monotonicity, subadditivity, positive homogeneity, and translation invariance are recent tools in risk management to assess the amount of risk agents are exposed to. If they also satisfy law invariance and comonotonic additivity, then we get a subclass of them: spectral measures of risk. Expected shortfall is a well-known spectral measure of risk.

We investigate the above mentioned six axioms using tools from general equilibrium (GE) theory. Coherent and spectral measures of risk are compared to the natural measure of risk derived from an exchange economy model, which we call the GE measure of risk. We prove that GE measures of risk are coherent measures of risk. We also show that spectral measures of risk are GE measures of risk only under stringent conditions, since spectral measures of risk do not take the regulated entity's relation to the market portfolio into account. To give more insights, we characterize the set of GE measures of risk via the pricing kernel property.

☆ We are grateful to two anonymous referees and conference and seminar participants in Maastricht, Amsterdam, Budapest and Lisbon for helpful comments.
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[1] The author would like to thank the Netherlands Organisation for Scientific Research (NWO) for financial support.
[2] The author would like to thank funding by the Netherlands Organisation for Scientific Research (NWO) and by the European Union under the Marie Curie Intra-European Fellowship MEIF-CT-2004-011537.

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doi:10.1016/j.jbankfin.2006.10.026

JEL classification: D51; G10; G12

Keywords: Coherent measures of risk; General equilibrium theory; Exchange economies; Asset pricing

1. Introduction

Risk management is of crucial importance considering the enormous financial risk our economy is exposed to. The risks of many economic agents are regulated by various institutions. For example, if a financial trader wants to sell options, which give the buyer rights of buying or selling at a given price during a specified time horizon (or at a given time), he has to fulfil margin requirements, i.e. he has to deposit some cash or some other riskless and liquid instrument. An exchange's clearing firm, which is responsible for the promises to all parties of transactions being securely completed, requires margin deposits. A measure of risk can be used to determine the margin requirement. The riskier the trader's portfolio, the more the margin requirement should be.

Other external regulators, at an international level, are the International Actuarial Association (IAA) and the International Accounting Standards Board (IASB), who determine the capital requirements for insurance companies. Similarly, the Basel Committee gives guidelines for the acceptable level of capital on banking supervision. Since a government or central bank could be a lender of last resort for these institutions, and the default of them could cause serious problems, they are regulated as well. As an internal regulator, a portfolio manager has to regulate the risk of its traders. In the context of a multi-division firm setting, the head-office may also set risk-limits for the divisions. Internally the risk values can also be used for planning and performance evaluation. It is therefore crucial to measure risk in an appropriate way.

We will use the term *portfolio* when referring to a risky entity (portfolio, firm, insurance company, bank, etc.). The value of a portfolio might change due to all kinds of uncertain events. We relate *risk* to the probability distribution of the future value of the portfolio. For the sake of simplicity in this paper we use discrete random variables. Our approach can be extended to the case of continuous risks and risks with unbounded support. All this requires is an analysis of competitive equilibrium in such an environment. The interested reader is referred to Chapter 10 of Duffie (2001).

A *measure of risk* assigns a real number to a random variable. It is the minimal amount of cash the regulated agent has to add to his portfolio, and to invest in a zero coupon bond. *Coherent measures of risk* (Artzner et al., 1999) are defined by four axioms: monotonicity, subadditivity, positive homogeneity and translation invariance. When adding two more axioms: law invariance and comonotonic additivity we get a subclass of coherent measures of risk, namely *spectral measures of risk* (Acerbi, 2002). Expected shortfall is a well-known spectral measure of risk (Acerbi and Tasche, 2002). For an introduction to risk measures and the above mentioned axioms see for instance Chapter 4 of Föllmer and Schied (2002).

Our approach is to model the situation at hand as an exchange economy in a general equilibrium (GE) setting, and determine which axioms are compatible with this model, and whether other axioms emerge as natural. This approach has the advantage that it recognizes the fact that the risk of a portfolio depends on the other assets present in the economy (the *market portfolio*), an insight that is generated immediately by the Capital Asset