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Two new operators in rough set theory with applications to fuzzy sets

Huaguang Zhang [a,*], Hongli Liang [a], Derong Liu [b]

[a] *School of Information Science and Engineering, Northeastern University, Shenyang, Liaoning 110004, PR China*

[b] *Department of Electrical and Computer Engineering (M/C 154), University of Illinois at Chicago, 851, South Morgan Street, Chicago, IL 60607-7053, USA*

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Abstract

In this paper, two new operators are introduced for the rough set theory. Using them, two inequalities well known in the rough set theory can now be modified to become equalities. With this change, no information will be lost in the new expressions. Hence, many properties in rough set theory can be improved and in particular, the union, the intersection, and the complement operations can be redefined based on the two equalities. Furthermore, the collection of rough sets of an approximation space forms a Boolean algebra under these new operators. Finally, roughness properties of fuzzy sets are analyzed using the new operations.

Keywords: Roughness of fuzzy sets; Rough set theory; Certain increment operator; Uncertain decrement operator; Boolean algebra

1. Introduction

In 1982, Pawlak introduced the rough set theory [7], which has emerged as another major mathematical tool for modelling the vagueness present in human

* Corresponding author. Fax: +86-24-8368-7762.
E-mail addresses: hg_zhang@21cn.com, z_hg@163.net (H. Zhang).

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classification mechanism. This concept is fundamental to the examination of granularity in knowledge [9,20,21]. It is a concept that has many applications in machine learning, pattern recognition, decision support systems, expert systems, data analysis, and data mining, among others. The theory of fuzzy sets introduced by Zadeh has provided a useful mathematical tool for describing the behavior of systems that are too complex or ill-defined to admit precise mathematical analysis by classical methods and tools. Extensive applications of the fuzzy set theory have been found in various fields. There have been many papers studying the connections and differences of fuzzy set theory and rough set theory [1–3,5,10–12]. One important example of such studies is the roughness of fuzzy sets where some notions of rough sets and fuzzy sets are integrated [1].

The R-lower and R-upper approximations of X are two basic concepts in rough set theory [14,15,19]. They have many properties that have become the foundation of rough set theory [13]. In particular the properties $\underline{R}(X \cup Y) \supseteq \underline{R}X \cup \underline{R}Y$ and $\overline{R}(X \cap Y) \subseteq \overline{R}X \cap \overline{R}Y$ are of great importance, for they do not allow for step by step computation of approximations. In other words, approximations of $X \cup Y$ cannot be done in general by the ones of X and of Y. These properties are a logical, formal consequence of the assumed definition of the knowledge base expressed in the form of approximations [8]. They have brought inconvenience and difficulties in many fields. The purpose of this paper is to show that these inequalities can be modified to become equalities by defining two new operators and use the two equalities to establish results for the algebraic operation of rough sets, the roughness of rough sets, and the roughness of fuzzy sets.

The paper is organized as follows. In Section 2, we discuss the basics of rough sets and provide some definitions. In Section 3, we define two new operators, the certain increment operator and the uncertain decrement operator, and we establish their properties. In Section 4, we define the operations in rough sets and we discuss the properties of these operations. In Section 5, we introduce the Boolean algebra of rough sets and we show that the collection of rough sets of an approximation space forms a Boolean algebra under the present new operations. In Section 6, we establish results for the roughness of fuzzy sets. In Section 7, we conclude the present paper.

2. Definitions and notation

We introduce in this section some definitions and notation used in the present paper.

Definition 2.1. Let U be a finite and non-empty set which is called the universe. Let R be an equivalence relation on U. We use U/R to denote the family of all equivalence classes of R (or classifications of U), and we use $[x]_R$ to denote an equivalence class in R containing an element $x \in U$.