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Linear and non-linear price decentralization

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Abstract

Compendious and thorough solutions to the existence of a linear price equilibrium problem, the second welfare theorem, and the limit theorem on the core are provided for exchange economies whose consumption sets are the positive cone of arbitrary ordered Fréchet spaces—dispensing entirely with the assumption that the vector ordering of the commodity space is a lattice. The motivation comes from economic applications showing the need to bring within the scope of equilibrium theory vector orderings that are not lattices, which arise in the typical model of portfolio trading with missing options. The assumptions are on the primitives of the model. They are bounds on the marginals of non-linear prices and for ω-proper economies they are both sufficient and necessary.

JEL classification: C61; C62; D46; D51; G1

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1. Introduction

The second half of the 20th century saw an explosion of interest in the Walrasian model of general equilibrium. Foundational results include the existence of at least one competitive equilibrium, the characterization of valuation equilibria as efficient allocations, and the characterization of competitive equilibria as Edgeworth equilibria.

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For several decades a primary research program in general equilibrium theory has been to establish the classical theorems on price decentralization and the price equilibrium existence problem in a context sufficiently general to encompass as particular instances the important general equilibrium models which arise in applications. This program was well articulated by Mas-Colell [33]. The idea is to separate the very difficult mathematical problems associated with price decentralization from their applications.

The principal contributors to this program are, of course, Mas-Colell [33] and Mas-Colell and Richard [34]. In those papers the authors replace the pervasive finite dimensional assumption on the interiority of endowments in consumption sets with two requirements:

(1) The commodity space is ordered by a vector lattice ordering that is compatible with the topology of the space.[1] The traders' consumption sets coincide with the positive cone of this ordering and preferences are monotone with respect to this ordering and continuous with respect to the topology of the commodity space.
(2) Preferences satisfy an assumption termed ω-uniform properness (where ω is the total endowment of resources), which is a cone condition that could in some settings be interpreted as a bound on the marginal rates of substitution (see for instance [18,12,38,29]).

The work of Mas-Colell [33] was extended in [3,4,16,17,43,44] and the work of Mas-Colell and Richard [34] was extended in [14,21,24,32,36,37,41,42]. In all these papers some variant of ω-properness is assumed and the lattice theoretic arguments are used in a non-trivial way. However, the use of lattice-theoretic techniques comes at a price: the very structure of the economy is expressed in terms of the vector lattice ordering of the commodity space. For instance, the constraints on consumption sets, the notion of monotone preferences, the properness assumptions, free disposal, the notion of a free disposal equilibrium, the topology of the commodity space, and even the compactness assumptions on the set of feasible allocations are all defined with respect to the vector lattice ordering of the commodity space. Therefore, the economic meaning of these results hinges on the interpretive efficacy of the ordering of the space.

Commodity spaces that are not lattice ordered arise naturally in many economic models and the large literature on price decentralization in vector lattices has little, that is obvious, to say in such a setting. An example of such an economic model is portfolio trading when markets are incomplete. It is known that in such models all the decentralization results can fail even if preferences are uniformly proper and the commodity space is finite dimensional. In these models consumers are motivated by the payoff of a portfolio. Therefore, the meaningful natural ordering of the portfolio space is the one that compares portfolio payoffs and which is closely related to the notion of first-order stochastic domination. In fact, the notion of *arbitrage free prices* is an order theoretic notion that induces this natural ordering of the portfolio space. Unfortunately, this ordering is rarely a vector lattice ordering when markets are not complete. The basic intuition for this is the following. Generally, when markets are not complete some call and put options cannot be replicated as the payoff of a portfolio of available securities. However, call and put options are closely related to the order structure of the portfolio space. Indeed, every marketed option is a lattice operation in the portfolio

[1] Mas-Colell assumes that the lattice operations are uniformly continuous and Mas-Colell–Richard assume that the topological dual of the commodity space is a Riesz subspace of the order dual.