Journal of Mathematical Behavior 31 (2012) 73–90

Contents lists available at SciVerse ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

The role of multiple solution tasks in developing knowledge and creativity in geometry

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ARTICLE INFO

Available online 30 November 2011

Keywords:
Multiple solution tasks
Mathematical creativity
Geometry knowledge

ABSTRACT

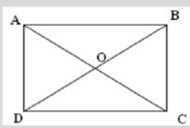
This paper describes changes in students' geometrical knowledge and their creativity associated with implementation of Multiple Solution Tasks (MSTs) in school geometry courses. Three hundred and three students from 14 geometry classes participated in the study, of whom 229 students from 11 classes learned in an experimental environment that employed MSTs while the rest learned without any special intervention in the course of one school year. This longitudinal study compares the development of knowledge and creativity between the experimental and control groups as reflected in students' written tests. Geometry knowledge was measured by the correctness and connectedness of the solutions presented. The criteria for creativity were: fluency, flexibility, and originality. The findings show that students' connectedness as well as their fluency and flexibility benefited from implementation of MSTs. The study supports the idea that originality is a more internal characteristic than fluency and flexibility, and therefore more related with creativity and less dynamic. Nevertheless, the MSTs approach provides greater opportunity for potentially creative students to present their creative products than conventional learning environment. Cluster analysis of the experimental group identified three clusters that correspond to three levels of student performance, according to the five measured criteria in pre- and post-tests, and showed that, with the exception of originality, performance in all three clusters generally improved on the various criteria.

1. Introduction

Nir is a 10th grade student, a high achiever in mathematics, who volunteered to be interviewed at the beginning of the study. He loves mathematics and enjoys solving problems. According to his teacher, "he is smart and curious." During the interview Nir was asked to solve two geometry problems, each in as many ways as he can. This requirement was new to him because usually he is asked to solve problems for which one solution is sufficient.

Problem 1

Prove that a parallelogram with equal diagonals is a rectangle.



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doi:10.1016/j.jmathb.2011.11.001

In his *first solution* Nir proved that triangle ADC is congruent with triangle BCD, and therefore $\angle ADC = \angle BCD$, and because of the parallel lines the sum of the two angles is 180°, and therefore it can be concluded that each angle is 90°. Nir completed the proof arguing that a parallelogram with a right angle is a rectangle.

Now Nir was asked to find another proof. At first he had difficulty disconnecting himself from his first proof and restarting without any conclusions derived from the first solution. He marked $\angle BCD$ as α and said:

1	Nir:	I have to prove that α equals 90°, but if this is α then all the angles are α
	Anat:	Why?
2	Nir:	$\angle BAD = \angle BCD$ and $\angle ADC = \angle BCD$ because these are opposite angles in a parallelogram and we already proved that $\angle ADC = \angle BCD$

The interviewer now asked Nir to find a new solution without relying on what has already been proved. Nir answered:

3	Nir:	**What else besides congruency?** . . . You mean something without congruency at all?
	Anat:	It is possible to solve without using triangle congruency or using a different one.
4	Nir:	So there is a possibility to solve it without congruency. [After 25 seconds he continued] but if the diagonals are equal how is it possible without congruency? . . . **Do I have to add a line** [auxiliary construction]?
	Anat:	Not necessarily.
5	Nir:	**I don't have any idea.** [After 33 seconds he continued] I know how to show that it is 90°. This is a median [points at CO] in triangle BCD because the diagonals of a parallelogram bisect each other.
	Anat:	So?
6	Nir:	Is there something [theorem] saying that if a **median equals half of the hypotenuse than it is a right triangle**?. . . I wasn't sure.

In this way, the *second solution* of the problem was based on the theorem that a triangle in which the median equals half the side it bisects is a right triangle. As his *third solution*, Nir proved that triangle AOB is congruent with DOC and AOD with BOC. But then he said:

7	Nir:	The truth is that **this is the same**.
	Anat:	The same as what?
8	Nir:	We already proved that the larger triangles are congruent.

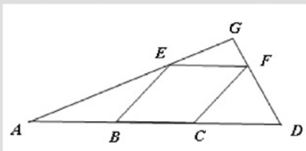
Anat showed Nir that by marking $\angle ACD$ as α and $\angle BCD$ as β, it was also possible to demonstrate, without using congruency, that the 4 triangles divide each of the parallelogram's angles into two angles that equal $\alpha + \beta$.

9	Nir:	Oh, they [the angles] are all equal. Nice! So it can be done this way or this way.

After the first problem was solved in four ways (one of which was performed with Anats' help) Nir was asked to solve Problem 2.

Problem 2

In triangle AGD, points E and F are on AG and DG respectively, and points B and C are on AD (see drawing). Given that EF=FC=CB=BE, prove that triangle AGD is a right triangle. (See Fig.1 for different solutions to the problem)



Nir's *first solution* involved computations of angles based on the property of rhombus EFBC. For the *second solution* he connected the problem to the theorem used in Problem 1.

10	Nir:	How do I get to 90°. . . without calculating angles? **It must be something to do with calculating angles unless there is again some theorem**. For 90°, a median to the hypotenuse doesn't help me here. Is there a different theorem, I mean except to calculate angles?

Eventually Nir solved the problem in three ways: (a) calculating angles based on the property of rhombus EFBC, (b) using the diagonals of rhombus EFBC and parallelograms EFDC and EFBA to show that $\angle EGF$ is an angle of a rectangle, and (c) drawing one of the rhombus diagonals and using the theorem that a triangle in which the median equals half the side it bisects is a right triangle. Clearly, he applied the strategy used in Problem 1 in Problem 2 in the two additional ways and was therefore able to prove that an angle is a right angle without calculating angles, which appears to be the students' natural strategy for such tasks.

The excerpts above demonstrate that students build easily their conceptions of what it means to solve a geometry problem in different ways, as well as reveal students' flexibility and the way they connect knowledge by implementing MSTs. Clearly, at the beginning of the interview Nir did not understand what was meant by a different way (e.g., [1]-[2]), but in the course of the interview he understood that a different way may be based on different tools (e.g., [3]-[6], [10]). He also developed understanding of the meta-mathematical notion of different solutions and sensitivity to the level of differences between solutions. Nir sensed that using repeatedly congruency of triangles is not as different as using various theorems (e.g., [7]-[8]). We also observed changes in Nir's flexibility in producing different solutions when he solved Problem 2 by reference to Problem 1. Whereas at the beginning Nir did not believe that there was more than one solution to Problem 1 ("but if the diagonals are equal, how is it possible without congruency," [3]-[4]), when addressing Problem 2 he immediately suggested that it can be solved not only by computing angles but also by using the theorem of the median to the hypotenuse. He also