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Percolation on shopping and cashback electronic commerce networks

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HIGHLIGHTS

- We study percolation properties of Shopping and Cashback Electronic Commerce Networks.
- We derive the exact expressions for the percolation threshold of SCECNs.
- We propose arithmetic to calculate the corresponding structural measures of SCECNs.
- We observe the accuracy of our arithmetic, and give explanations on the discrepancies.
- We show those structural measures are useful to appraise the status of SCECNs.

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ABSTRACT

Many realistic networks live in the form of multiple networks, including interacting networks and interdependent networks. Here we study percolation properties of a special kind of interacting networks, namely Shopping and Cashback Electronic Commerce Networks (SCECNs). We investigate two actual SCECNs to extract their structural properties, and develop a mathematical framework based on generating functions for analyzing directed interacting networks. Then we derive the necessary and sufficient condition for the absence of the system-wide giant in- and out- component, and propose arithmetic to calculate the corresponding structural measures in the sub-critical and supercritical regimes. We apply our mathematical framework and arithmetic to those two actual SCECNs to observe its accuracy, and give some explanations on the discrepancies. We show those structural measures based on our mathematical framework and arithmetic are useful to appraise the status of SCECNs. We also find that the supercritical regime of the whole network is maintained mainly by hyperlinks between different kinds of websites, while those hyperlinks between the same kinds of websites can only enlarge the sizes of in-components and out-components.

1. Introduction

Numerous economic, social, biological and technological systems can be properly described as complex networks, which have been studied intensively in the past decade. In this field, most results have been obtained by modeling and analyzing isolated networks, while interactions between these networks were totally ignored. Recently, this defect has been revealed by several papers [1–3]. They pointed out that isolated network rarely occurs in reality, instead multiple networks with distinct topologies may coexist and compose a larger network system [1], meanwhile these individual networks are interconnected and events taking place in one are likely to affect the others [2,3]. Therefore, investigating and analyzing the

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connectivity properties and structural disciplines of complex networks under the circumstances of ''networks of networks'' is of academic value and practical meaning.

So far, models focusing on ''networks of networks'' present some diversity, and several representatives are worthy to be scrutinized. Perhaps the most attractive and well known ones in this field are those talking about interdependent networks [2–5]. These models emphasized that modern networks are coupled in such a way that the failure of nodes in one network may lead to the failure of dependent nodes in other networks, which in turn causes further damage to the first network, leading to cascading failures as well as catastrophic consequences. Meanwhile, these models usually contain both dependency links which coin the relation between two nodes depending on each other, and connectivity links which denote that there exists only a communication channel between a pair of nodes [6]. In comparison with them, another influential model proposed by Leicht and D'Souza [1] concentrated on undirected interacting networks containing connectivity links only. They developed a mathematical framework based on generating functions for analyzing these interacting networks given the node connectivity within and between networks, and derived exact expressions for the percolation threshold. So their model also extended the application of generating functions into the area of ''networks of networks''. Except models described above, there is a class of models seizing the fact that an agent may play different roles in different networks at the same time, thereby one node can appear in more than one network simultaneously, examples are Szell et al. [7], and Yagan et al. [8]. Their analyses always involved both every separate individual network and the conjoint network composed of all these individual networks aiming at finding and exploring some interesting properties belonging to the conjoint network.

On one hand, these models provide us with more facilities to analyze and appraise realistic multiple networks system, and help give some reasonable development proposals. On the other hand, many realistic multiple networks systems have particular structural characteristics, which spur us to append more contents as well as constraint conditions to the primary model. This kind of work will improve the explanatory capability of the primary model, and make the theory of multiple networks system more close to reality.

In this study, we mainly focus on percolation properties of a special kind of electronic commerce networks, namely Shopping and Cashback Electronic Commerce Networks (SCECNs). As usual, we define websites as nodes and hyperlinks as edges. Nodes of SCECNs can be divided into shopping websites and cashback websites, and each website does not depend on other websites so much. If one website breaks down, other websites will still work with an invalid hyperlink to that website. So SCECNs are interacting networks containing connectivity links only, and it is appropriate to choose the model created by Leicht and D'Souza [1] as the base model. Unfortunately, this model can deal with undirected graph only, but we have to take edge directions into consideration. Two kinds of components draw our special attention, including the set of nodes which can be reached from a given node as well as the set of nodes from which a given node can be reached. The former determines the amount of commodity information and cashback information received by online consumers through hyperlinks within SCECNs, while the latter has the connection with the profitability of both kinds of websites. Further, conditions for these components to become the giant ones are also worthy to be discussed.

The rest of this paper is organized as follows. In Section 2, we give degree descriptions of two typical SCECNs, and extract structural properties of SCECNs from them. Based on these structural properties, in Section 3, we propose our mathematical framework for SCECNs, which is helpful to derive percolation conditions, and calculate the average sizes of those two kinds of components in the sub-critical regime, as well as the probabilities that a randomly chosen node is part of different giant components in the supercritical regime. In Section 4, we apply our arithmetic to those two SCECNs mentioned in Section 2 to observe its accuracy, and use analytical values to appraise the status of SCECN 1 as well as contributions of different kinds of hyperlinks. Finally, we present a discussion and conclusion in Section 5.

2. Structural properties of SCECNs

As we mentioned earlier, a SCECN is composed of shopping websites and cashback websites. Shopping websites are very familiar to us. These websites enable us to browse and purchase lots of commodities online, and some of them sell nearly all kinds of products such as Amazon, which sells books, electronics & computers, clothing, shoes & jewelry, automotive & industrial products, and so on. While others only provide a singular range of products such as ProFlowers which only sells flowers, and UGG Australia only sells shoes. Cashback websites are reward websites that will give a percentage of payments back to their members, if they purchase goods and services via affiliate links from these cashback websites to shopping websites. For example, Ebates is a famous cashback website in America. If a consumer want to buy flowers, he will visit www.ebates.com, and then through hyperlinks go to ProFlowers, after the transaction, he will get 16% cashback on each order from Ebates. Some derivative websites of these cashback websites may give points to their members instead, and these points can be used to exchange money or some commodities all by their members' wishes.

No matter which reward form these cashback websites adopt, their earnings as shares of their members' payments mainly come from shopping websites. So cashback websites usually want to sign cashback contracts with shopping websites, and they try to place many hyperlinks to the same type of shopping websites altogether to increase purchase opportunities for their members. Ebates admits that it receives a fee for referring buyers to affiliated stores and uses that fee to pay members cashback on their purchases. It allows its members to earn cashback on purchases made at over 1500 stores, and these stores have been categorized into different labels based on their commodity range. This design offers online consumers a map of shopping websites providing commodities of interest for selection aside from their purchase rewards. All of these have caused cashback websites to quickly develop since the middle of the 2000s all over the world.