Physica A 435 (2015) 89–94

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Universal laws of human society's income distribution☆

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HIGHLIGHTS

- We translate procedure of statistical physics into language of neoclassical economics.
- General equilibrium equations play the same role with many-body Newtonian equations.
- General equilibrium in economics plays the same role with microstate in physics.
- Rawls' principle of social fairness approves hypothesis of equal probability.
- Method of statistical physics can reproduce standard result of neoclassical economics.

ARTICLE INFO

Article history:
Received 11 August 2014
Received in revised form 27 March 2015
Available online 9 May 2015

Keywords:
General equilibrium
Arrow's Impossibility Theorem
Hypothesis of equal probability
Rawls' fairness
Entropy

ABSTRACT

General equilibrium equations in economics play the same role with many-body Newtonian equations in physics. Accordingly, each solution of the general equilibrium equations can be regarded as a possible microstate of the economic system. Since Arrow's Impossibility Theorem and Rawls' principle of social fairness will provide a powerful support for the hypothesis of equal probability, then the principle of maximum entropy is available in a just and equilibrium economy so that an income distribution will occur spontaneously (with the largest probability). Remarkably, some scholars have observed such an income distribution in some democratic countries, e.g. USA. This result implies that the hypothesis of equal probability may be only suitable for some "fair" systems (economic or physical systems). From this meaning, the non-equilibrium systems may be "unfair" so that the hypothesis of equal probability is unavailable.

1. Introduction

Statistical physics was extraordinarily successful in dealing with the physical properties of systems which consist of huge numbers of particles. Although the number of economic agents in an economic system is less than the counterpart quantity in physical system, there are still 10^6 firms and 10^7 households, and these are large numbers [1]. Because of this, the statistical physics was also applied in studying the statistical properties of complex economic systems consisting of a large number of economic agents [1–12]. It was widely accepted that [6] the data analysis of empirical distributions of income reveals a two-class distribution. As is well known, Yakovenko [6] has used the income data from 1983 to 2000 in USA to confirm that the majority of the population (lower class) obey Boltzmann distribution (i.e., exponential distribution) and the small fraction of the population (upper class) obey Pareto distribution (i.e., power-law); see Fig. 6 in Ref. [6].

☆ Project supported by the Fundamental Research Funds for the Central Universities (Grant No. SWU1409444); The National Social Science Foundation of Great Invite Public Bidding Project (Grant No. 11&ZD047); The National Soft Science Research Plan (Grant No. 2013GXS4D143); The Fundamental Research Funds for the Central Universities (Grant No. SWU1309369).

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http://dx.doi.org/10.1016/j.physa.2015.05.001
0378-4371/

Nonetheless, recently, Kusmartsev et al. [8,9] used the income data from 1996 to 2008 in USA to find that the majority of the population (lower class) abnormally obey Bose–Einstein distribution but the small fraction of population (upper class) still obeys Pareto distribution; see Fig. 1 in Ref. [8]. Thus, Yakovenko's and Kusmartsev's studies seemed to contradict the agreement that the lower class of population will obey the universal law. However, when Tao [13] applied the principle of maximum entropy into Arrow–Debreu's general equilibrium model, he found that the equilibrium income distribution among social members will obey Boltzmann distribution for a moderately competitive society but will obey Bose–Einstein distribution for an extremely competitive society. In particular, the economic crisis (e.g. financial crisis in 2008) can be just regarded as a Bose–Einstein condensation among social members [7]. Thus, Tao's results not only explain why Yakovenko's and Kusmartsev's research results are different in investigating the lower class of population, but also further imply that the lower class of population well occupy the general equilibrium state, and that the upper class may be far from the general equilibrium. From this meaning, Tao indeed provided an alternative theoretical explanation to clarify the universal law of humans' income distribution. On the basis of Tao's previous work [13], this paper will translate the standard procedure of statistical physics into the standard language of neoclassical economics. We hope that such an attempt can serve some communications between physicists and economists.

The main purpose of statistical physics is to obtain the most probable macrostate which will contain the most microstates. In general, each microstate is specified by a collection of generalized coordinates and generalized momentums:

$$(q_1, \ldots, q_D; p_1, \ldots, p_D)\,, \tag{1}$$

where q_i and p_i denote the generalized coordinate and the generalized momentum of the ith particle respectively.

Since each particle must obey the Newtonian equations, each q_i and p_i will be determined by the Hamiltonian equations,[1] which are on the basis of the Principle of Least Action. As such, we immediately comprehend that each microstate is essentially a solution of the D-body Newtonian equations at a given time. To obtain the most probable macrostate, the key step is to assume that every microstate will occur with an equal probability.

Similar to the physical system in which the behaviors of particles must obey the Newtonian equations, the optimal behaviors of social members in the economic system will be governed by Arrow–Debreu's general equilibrium equations (ADGEE) which are the cornerstone of neoclassical economics [14]. In neoclassical economics, ADGEE is derived by Rational Person Hypothesis, which is the fundamental economic principle. It is here interesting to point out that the Principle of Least Action in physics is also thought of as Nature's own principle of economy.

The solution to ADGEE is called the competitive equilibrium which corresponds to an optimal income allocation. Similar to many-body Newtonian equations, ADGEE will have multiple solutions (for a long time). Then by Arrow's Impossibility Theorem social members will face the "dilemma of social choice". This dilemma implies that the best income allocation cannot be found. Interestingly, this paper shows that the principle of maximum entropy will provide a possible way of removing the "dilemma of social choice" in a just society which obeys Rawls' fairness. Later, we shall see that each solution to ADGEE can be regarded as a possible microstate of the economic system, so that the principle of maximum entropy may be available in a society which captures Rawls' fairness.

2. Arrow–Debreu's general equilibrium model

Following the standard framework of neoclassical economics [14], we assume that there are N consumers, N firms and L types of commodities. Every consumer $i = 1, \ldots, N$ is specified by a consumption set $X_i \subset R^L$, a preference relation $\underset{\sim i}{\succ}$ on X_i, an initial endowment vector $\omega_i \in R^L$. Each firm $j = 1, \ldots, N$ is characterized by a production set $Y_j \subset R^L$. We denote by $x_i = (x_{1i}, \ldots, x_{Li})$ the consumption vector of the ith consumer, where $x_i \in X_i$ and $x_{ki} \geq 0$ for $k = 1, \ldots, L$. We denote by $y_j = \left(y_{1j}, \ldots, y_{Lj}\right)$ the production vector of the jth firm, where $y_j \in Y_j$. Adopting the input–output model's convention [15], y_j's positive component denotes output and negative component denotes input. Without loss of generality, we assume that all the firms only produce the mth type of commodity, namely $y_{mj} \geq 0$ for $j = 1, \ldots, N$ and $y_{lj} \leq 0$ for $l \neq m$. The economic meaning of this assumption is that there is one industry only. Such an assumption will help us to simplify the complexity of calculation, and does not influence our final result; for details see Tao's [13] discussion about multiple industries.

Because our main purpose[2] is to investigate income allocation and income distribution among social members, we further assume that the ith consumer is the owner of the ith firm, where $i = 1, \ldots, N$. Thus, the revenue of the ith firm is identified with the income of the ith consumer.[3] It is carefully noted that "income" in this paper is somewhat different from "wealth". For instance, "wealth" may be due to either property inheritance or individual effort, but "income" in this paper only involves individual effort.

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1 That is, $\begin{cases} \dfrac{\mathrm{d}q_i}{\mathrm{d}t} = \dfrac{\partial H}{\partial p_i} \\ \dfrac{\mathrm{d}p_i}{\mathrm{d}t} = -\dfrac{\partial H}{\partial q_i} \end{cases}$ for $i = 1, \ldots, D$.

2 Since national income differences may be the biggest problem facing the world today [16], our attention will be concentrated on the income distribution among social members.

3 From the empirical point of view, these entrepreneurs can be regarded as a sampling about income allocation.