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Learning and adaptation's impact on market efficiency☆

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ABSTRACT

A dynamic model with learning and adaptation captures the evolution in trader beliefs and trading strategies. Through a process of learning and observation, traders improve their understanding of the market. Traders also engage in a process of adaptation by switching between trading strategies based on past performance. The asymptotic properties are derived analytically, demonstrating that convergence to efficiency depends on the model of adaptation.

1. Introduction

Financial markets offer considerable evidence suggesting traders do not have a full and complete understanding of the process by which prices are determined. There also seems to be a lack of consensus among traders in interpreting what the current price signals about future payoffs. Evidence of disagreement includes, but is not limited to, excess volatility, high trading volume, the considerable variety in trading strategies, and the fervor with which traders seek to improve their models. This suggests that markets have yet to achieve the rational expectations equilibrium described by Grossman and Stiglitz (1980). Instead, traders appear to process information, develop strategies, and adapt to changing market conditions.

To gain insight into the impact of evolution in trading strategies on financial markets, this paper recasts Grossman and Stiglitz (GS) as a fully specified dynamic model. The absence of a rational expectations equilibrium in the presence of a revealing price ensures that there is always room for improvement in trading strategy. The two dynamics, learning and strategy adoption, capture two aspects of trader adaptation to the observed market environment. The single period terminal asset of GS offers tractability in a sufficiently rich environment to examine the key elements of adaptation. The asymptotic properties of the market governed by these two dynamic process are developed herein.

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The GS model places informed traders in the market with rational uninformed traders. The former gain access to private information through costly research. The latter seek to extract the private information from the price. Each trader makes full use of the information to which he or she has access. This project retains much of the structure of the GS market as well as the notion that traders make full use of the available information.[1] In this behavior, they are rational, but here the uninformed traders are boundedly rational in that they are denied knowledge of the true relationship between the observed price and the expected payoff. The traders instead must, as in Bray (1982), attempt to learn the relationship through observation of market data. This is the learning process that GS presume to have already taken place prior to their analysis.[2] Distinct from Bray (1982), concurrent with the learning process, the traders choose between the two information options. Such a dynamic population is implicit in the GS model, but is not explicitly modeled in their static examination.[3] GS consider the fixed point equilibrium of the population process, having presumed prior convergence to rational expectations in the traders' beliefs. The importance of examining both as simultaneous processes is that the two dynamic processes, learning and the population process, interact, affecting market behavior both during evolution and in the asymptotic convergence.

Two types of population processes are examined and compared to explore the different implications for how they shape the market's asymptotic behavior. One family includes the Discrete Choice Dynamics introduced by Brock and Hommes (1997, 1998) and the other family includes Replicator Dynamics. Both processes have received extensive attention in the economics literature as tools for modeling evolving populations in a discrete choice setting.

The second key alteration to the GS model is to remove the random supply of the risky security, thereby removing the mechanism that ensures the existence of an equilibrium to the population process. As a result, the information advantage held by the informed traders only exists while the learning process is ongoing. Just how the dynamic model resolves the conflict between convergence in the learning process and the market's need for noise in the price offers insight into the evolution that results from traders' adaptation to endogenously changing market conditions. The resolution depends on the interaction between the two dynamic processes as it is shaped by the potential absence of the population fixed point. The nature of this interaction determines whether market efficiency is the limiting case.

The tractability of the present model is absent in the Goldbaum (2005) and Goldbaum (2006) simulations, also based on models of learning and strategy adoption. This examination's simpler market, partially the result of the terminal risky asset, enables analytical investigation of the relevant dynamic processes, eliminating the dependence on simulation based analysis.

The paper proceeds as follows. Section 2 develops the model and establishes the conditions for existence and stability of the rational expectations equilibrium as the fixed point to the dynamic processes. Section 3 considers the issue of market efficiency analytically and in simulation. Section 4 uses simulation to determine how the rate of learning is affected by the population process. Section 5 concludes.

2. Model and fixed point equilibrium

2.1. Market

Adopting a repeated GS framework, in each period a population of $N = N^I + N^U$ informed (I) and uninformed (U) traders trade a risky asset and a risk-free bond. The risk-free bond, with a price of one, pays R at the end of the period. The risky asset is purchased at the market determined price, p_t. At the end of the period it pays a randomly determined terminal value, u_t.

$$u_t = \bar{u} + \theta_t + \varepsilon_t \tag{1}$$

with, $\theta_t \sim N(0, \sigma_\theta^2)$, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, and $\text{cov}(\theta_t, \varepsilon_t) = 0$.

The determinates of u_t are IID. This process is known to the traders.

In each period, each trader maximizes a negative exponential utility function of end of period wealth. Under the assumption of normality in returns, the resulting demand for the risky asset is

$$q_{it}(p_t) = (E_{it}(u_t) - Rp_t)\gamma\phi_{it} \tag{2}$$

with $\phi_{it} = 1/\sigma_{it}^2$. Here, $1/\gamma$ is the coefficient of absolute risk aversion and $E_{it}(u_t) = E(u_t|F_{i,t})$ and $\sigma_{it}^2 = \text{var}(u_t|F_{i,t})$ are trader i's conditional expectations and variance, respectively, of his or her forecast error.

Define $\lambda_t = N_t^I/N$ to be the proportion of informed traders, leaving $(1 - \lambda_t)$ as the proportion of uninformed traders. Let q_t^k be the per capita demand for the risky security among group $k = I, U$ traders in period t. In a Walrasian equilibrium, the market price equates supply and demand for the risky asset. The supply is fixed, avoiding the introduction of exogenous

[1] This is in contrast to other models of multiple trader types with switching in which the market-based traders are limited in their effort to extract information from past prices rather than from contemporaneous information, as employed in Brock and Hommes (1998), Chiarella and He (2001), Föllmer et al. (2005), and Gaunersdorfer et al. (2008), among others.

[2] "They [traders] learn the relationship between the distribution of return and the price, and use this in deriving their demand for the risky assets" (p. 394).

[3] "We can calculate the expected utility of the informed and the expect utility of the uninformed. If the former is greater than the latter (taking account of the cost of information), some individuals switch from being uninformed to being informed (and conversely) (p. 394).