Nash equilibrium strategies for a defined contribution pension management

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**HIGHLIGHTS**

- A time-consistent defined contribution pension scheme is considered.
- The problem is formulated as a bi-objective stochastic problem of mean–variance.
- The inflation risk and the salary risk are taken into account.
- Analytical expressions for the time-consistent strategies and value function are obtained.
- Interesting properties of the time-consistent results under inflation are analyzed in detail.

**ARTICLE INFO**

Article history:
Received March 2014
Received in revised form March 2015
Accepted 21 March 2015
Available online 1 April 2015

Keywords:
Defined contribution
Stochastic inflation
Stochastic salary
Nash equilibrium strategy
Mean–variance

**ABSTRACT**

This paper studies the time-consistent investment strategy for a defined contribution (DC) pension plan under the mean–variance criterion. Since the time horizon of a pension fund management problem is relatively long, two background risks are taken into account: the inflation risk and the salary risk. Meanwhile, there is a risk-free asset, a stock and an inflation-indexed bond available in the financial market. The extended Hamilton–Jacobi–Bellman (HJB for short) equation of the equilibrium value function and the verification theorem corresponding to our problem are presented. The closed-form time-consistent investment strategy and the equilibrium efficient frontier are obtained by stochastic control technique. The effects of the inflation and stochastic income on the equilibrium strategy and the equilibrium efficient frontier are illustrated by mathematical and numerical analysis. Finally, we compare in detail the time-consistent results in our paper with the pre-commitment one and find the distinct properties of these two results.

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1. Introduction

Nowadays it is well-known that the defined contribution plans have received more attention than the defined benefit plans and have become the dominant form of pension plans all over the world in recent years. The reason for this phenomenon lies in the recognized fact that the aging population problem is threatening the sustainability of a sufficient retirement income for the pensioners, and the defined contribution plan has an advantage over the defined benefit one to ease the press of public financial system by transferring the investment risk from the sponsor to the retiree. In a defined contribution plan, the plan member continuously contributes a fixed proportion of his stochastic salary to the pension fund, and then the contributions are invested in some suitable assets available in the market. Only the contributions to the account are guaranteed while the future benefits fluctuate on the basis of investment earning. Moreover, the DC pension fund investment management usually considers a long time horizon, and the benefits will be obtained nearly on retirement. Therefore, usually one will take broad categories of risks into account, such as the investment risk, the stochastic salary risk and the inflation risk. In the past years, the optimal investment problem in DC pension fund management has been extensively studied. However, an overwhelming amount of literature has focused on maximizing the expected utility of the terminal wealth under the CRRA or CARA criterion. See, for example, Boulier et al. (2001), Battocchio and Menoncin (2004), Cairns et al. (2006), Zhang et al. (2007), Gao
As mentioned above, in the past years, most of the existing DC pension fund management problems are based on the expected utility maximization, while there is a delay in solving the relevant problem under mean–variance framework. One reason for this delay, as Vigna (2014) said, lies in the fact that the multi-period and continuous-time versions of mean–variance problem have been produced only quite recently. But since the pioneering work of Markowitz (1952), the mean–variance analysis framework has been the foundation for modern portfolio selection theory; therefore, there appear several investigations on the DC pension fund management under the mean–variance criterion in recent years. Højgaard and Vigna (2007) solve a mean–variance DC management problem for the two-asset case as well as the long-term, Højgaard and Vigna (2007) solve a mean–variance DC management problem. Han and Hung (2012) and Blake et al. (2013) have been produced only quite recently. But since the pioneering work of Markowitz (1952), the mean–variance analysis framework has been the foundation for modern portfolio selection theory; therefore, there appear several investigations on the DC pension fund management under the mean–variance criterion in recent years. Højgaard and Vigna (2007) solve a mean–variance DC management problem for the two-asset case as well as the long-term, Højgaard and Vigna (2007) solve a mean–variance DC management problem. Han and Hung (2012) and Blake et al. (2013) have been produced only quite recently. But since the pioneering work of Markowitz (1952), the mean–variance analysis framework has been the foundation for modern portfolio selection theory; therefore, there appear several investigations on the DC pension fund management under the mean–variance criterion in recent years. Højgaard and Vigna (2007) solve a mean–variance DC management problem for the two-asset case as well as the long-term.

The reminder of this paper is organized as follows. Section 2 formulates the DC pension fund management problem under the mean–variance criterion. In Section 3, the associated extended HJB equation and the verification theorem are presented. Section 4 gives the equilibrium strategy and the equilibrium value function. Section 5 analyzes the properties of the equilibrium strategy and the equilibrium efficient frontier. The numerical results are also argued in this section. In Section 6, we compare our results with the case of pre-commitment. Section 7 concludes this paper. Proofs of lemmas and theorems are given in Appendices A–C.

2. Problem formulation

As mentioned in Section 1, since the time horizon of the pension management is usually very long, we cannot neglect the effects of the inflation. For simplicity, the (commodity) price index in this paper is assumed to follow the diffusion process:

\[ dI(t) = I(t)[\mu_1(t)dt + \sigma_I(t)dW(t)] \]

\[ I(0) = I_0, \]

where \( \mu_1(t) \) is the instantaneous expected rate of inflation, \( \sigma_I(t) \) is the volatility rate of the price index level, and \( W(t) \) is a standard one-dimension Brownian motion. The price index presents the price for a fixed basket of goods.
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