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A bi-level decision support system for uncertain network design with equilibrium flow

Suh-Wen Chiou *

Department of Information Management, National Dong Hwa University, Da Hsueh Rd., Shou-Feng, Hualien 97401, Taiwan

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ABSTRACT

A bi-level decision support system (BDSS) is proposed for a normative road network design with uncertain travel demand. A bi-level decision support model with link capacity expansion is developed to simultaneously reduce travel delay to road users and mitigate vulnerability of road network. A tractable solution scheme for BDSS is developed. Due to some hierarchy in decision-making order of BDSS, a bi-level programming is employed. A risk-averse Stackelberg solution is established for a normative BDSS under travel demand uncertainty. Numerical computations are performed using a real-data road network. Computational results indicate that the proposed solution scheme can effectively improve a worst-case performance of BDSS with greater success while incurring a relatively slighter loss of optimality when compared to deterministic solutions at nominal condition. Particularly, our computation results showed that proposed solution becomes more attractive as the realization taken by unknown demand growth factor increases.

1. Introduction

For most urban traffic road networks, severe travel delays could be incurred by road travelers as a result of insufficient provision of link capacity in the presence of travel demand surges and disruptive events. In this paper, a bi-level decision support system (BDSS) is proposed to cope with continuously growing travel demand and alleviate increasing traffic congestion via link capacity expansion. For a road network with uncertain travel demand, decision maker at upper level determines link capacity expansion with an objective of minimizing total travel time constrained by investment budget. The road users at lower level are supposed to minimize their journey travel time [1] from pairs of origin to destination through route choice for realization taken by unknown travel demand which is most unfavorable. The solution for the decision maker with precedence in decision can be regarded as a Stackelberg solution. The Stackelberg solution is an optimal strategy for the leader when road users react by playing optimally. A worst-case analysis is considered for a normative BDSS in the presence of unknown travel demand to mitigate vulnerability of road network. In this regard, a bi-level model is proposed for BDSS in order to effectively characterize a risk-averse Stackelberg equilibrium at worst case scenario.

In the presence of uncertainty there has been a growing number of research papers [2–8] investigating the performance reliability of networked system over past years. For instance, a spatial decision support system (SDSS) is developed in [5] to widely explore and examine the effects of different networked disruption scenarios. The proposed SDSS is helpful for decision makers in conveniently identifying critical network components and facilitate decisions about maintaining and enhancing network survivability. [7] also developed a network equilibrium model accounting for multi-criteria decision making behavior of various market participants for optimal pricing and resource allocation in a computational grid network. More recently, Burgholzer et al. [4] proposed a simulation tool for transportation network planners to support time-efficient alternative route choice of carriers in intermodal transportation networks in case of disruption. Considering a network design with uncertain input data, there are two mostly commonly used approaches in literature: stochastic programming and robust optimization [9]. From the prospective of stochastic programming, given a known priori distribution of probabilities of uncertain data, there are a variety of approaches applied to road network design with uncertain demand [10–13]. A growing interest in robust optimization approach [14–23] has attracted various applications. For example, a scenario-based robust solution in [14] is presented for large-scaled network design in which a stochastic linear programming approach is employed ([18–20]). Assuming uncertain data bounded within some certain set, there are plenty research works ([21–23]) extending a robust optimization ([15–17]) to road network design with uncertain demand. However, regarding a road network with equilibrium flow in the presence of uncertain travel demand, to the best of author's knowledge there is very limited research work using a bi-level programming approach to tackle a hierarchical decision making problem. For a general bi-level problem, decision variables at the upper level are optimized subject to the solution of lower level problem. As is noted by [24–27], in most cases a solution of lower level problem is not mathematically explicit.

* Tel.: +886 3 8633102.
E-mail address: chiou@mail.ndhu.edu.tw.

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A bi-level program generally turns out to be a non-convex problem and computationally intractable. Because of the non-convexity, solution algorithms in [28–31] can simply solve a bi-level problem of modest-size only locally. As noted from literature in [32–34], the equilibrium flow at lower level is generally not differentiable at some point. The first-order approximation for equilibrium flow may fail at these points. Therefore it would be preventive from direct use of the results in [35] for equilibrium flow. In this paper, we propose a novel and computationally tractable solution scheme based on recent work in sensitivity for generalized gradients [36–38] to solve BDSS in the presence of uncertain travel demand for equilibrium flow.

The contributions made from this paper are summarized as follows. Firstly, a bi-level decision support system (BDSS) is presented to determine optimal link capacity expansion for uncertain road network with equilibrium flow. A risk-averse Stackelberg equilibrium for a worst case scenario of system performance is established. The performance measure, maximized with respect to travel demand growth factor on the one hand, is minimized with respect to link capacity expansion in the presence of uncertain travel demand, on the other hand. In this regard, the worst-case performance measure serves as an upper bound estimate for link capacity expansion in the presence of a worst case travel demand. Secondly, a computationally tractable solution scheme is proposed for BDSS. To this end, a modified gradient-based approach using generalized gradients is presented. Thirdly, numerical computations are performed using a benchmark real-data road network with various initial data. The rest of the paper is organized as follows. Section 2 introduces a bi-level decision support system with a min–max model for equilibrium network flow. A bi-level programming approach is proposed. A risk-averse Stackelberg solution is characterized by a tractable computation scheme proposed in Section 3. Numerical computations are performed in Section 4 using a medium-size real-data road network with link capacity expansions. Conclusions for this paper and extensions of the proposed approach to topics of interest are briefly summarized in Section 5.

2. A BDSS problem formulation

A BDSS program is introduced for uncertain road network design with equilibrium flow. In the presence of uncertain travel demand, a BDSS with link capacity expansion can be regarded as a Stackelberg game. Both the decision maker with the leader at the upper level and road users with the followers at the lower level are trying to realize a best solution on their own with respect to some certain but generally different objectives. At the upper level the decision maker has the leadership in playing the game and can determine a set of robust link capacity expansions. The route choice chosen by users at the lower level for a worst-case realization taken by unknown demand strongly relies on link capacity expansion determined by decision maker at the upper level. That is, road users have to react optimally on decision maker's choice for a worst-case realization taken by unknown travel demand.

In the presence of uncertain demand, the solution for BDSS with equilibrium flow is considered as a risk-averse Stackelberg equilibrium. The constraints at the lower level can be defined in part by a parametric variational inequality. Notation used for a BDSS with respect to link capacity expansion under uncertain travel demand is summarized first.

2.1. Notation

$G(N, L)$	a road network with node set N and link set L.
W	a set of origin-destination (OD) pairs.
R_w	a set of routes between OD pair w, $\forall\, w \in W$.
q	a matrix of travel demand for OD pairs.
μ	a set of OD demand growth factor, $\mu = [\mu_w]$, $\forall\, w \in W$.
k	a vector of link current capacity, $k = [k_a]$, $\forall\, a \in L$.
u	a vector of link capacity expansion upper bound, $u = [u_a]$, $\forall\, a \in L$.
y	a vector of link capacity expansion, $y = [y_a]$, $\forall\, a \in L$.
f	a vector of average link flow, $f = [f_a]$, $\forall\, a \in L$.
h	vector of route flow between points of entry to points of exit from network, $h = [h_p]$, $\forall\, p \in R_w$, $\forall\, w \in W$.
λ	a link-route incidence matrix.
Λ	a OD-route incidence matrix.
$c(y, f)$	a vector of link flow travel cost, $c = [c_a(y_a, f)]$, $\forall\, a \in L$.
π	a vector of minimum travel cost between OD pair w, $\forall\, w \in W$, $\pi = [\pi_w]$.
C	a vector of route flow travel cost, $C = [C_p]$, $\forall\, p \in R_w$, $\forall\, w \in W$.
$V(y)$	a vector of link capacity expansion investment cost, $V(y) = [V_a(y_a)]$, $\forall\, a \in L$.
ω	a conversion factor from investment cost to travel cost.

2.2. A lower level problem

A user equilibrium flow at the lower level in BDSS can be identified by a variational inequality as follows. Let K denote a feasible set for network flow, i.e.

$$K = \{f : f = \lambda h, \Lambda h = q, h \geq 0\}. \tag{1}$$

According to [39], a user equilibrium flow can be characterized if and only if for every $\bar{f} \in K$ there exists a $f \in K$ such that

$$c(f)\left(\bar{f} - f\right) \geq 0. \tag{2}$$

For a BDSS with a set of link capacity expansion y in the presence of a realization taken by unknown demand growth factor μ, a responding user equilibrium flow $f(\mu, y)$ can be characterized in the following way. Let $K(\mu)$ denote a feasible set for a parametric user equilibrium flow with respect to some realization taken by unknown future demand growth factor μ, we have

$$K(\mu) = \{f : f = \lambda h, \Lambda h = \mu q, h \geq 0\}. \tag{3}$$

Therefore a parametric user equilibrium flow with demand growth factor μ can be characterized if and only if for every $\bar{f} \in K(\mu)$ there exists a $f(\mu, y) \in K(\mu)$ such that

$$c(y, f)\left(\bar{f} - f\right) \geq 0. \tag{4}$$

Let $\Omega_f(\mu, y)$ denote a solution set determined by Eq. (4) consisting of responding flow $f(\mu, y)$ to a set of link capacity expansion y, i.e.

$$f(\mu, y) \in \Omega_f(\mu, y). \tag{5}$$

Both link capacity expansion y^* and travel demand growth factor μ^* can be determined by a bi-level program such that a pair of saddle points (μ^*, y^*) exists. Let $Z_0(\mu, y, f)$ denote an objective function for a BDSS with equilibrium flow f. A Stackelberg solution (μ^*, y^*) is a saddle point if the following condition holds:

$$Z_0(\mu, y^*, f(\mu, y^*)) \leq Z_0(\mu^*, y^*, f^*) \leq Z_0(\mu^*, y, f(\mu^*, y)). \tag{6}$$

In Eq. (5), a responding user equilibrium flow f^* to a pair of saddle points (μ^*, y^*) is a solution of a parametric variational inequality Eq. (4). Therefore, we have

$$f^* = f(\mu^*, y^*). \tag{7}$$