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An EOQ model with backorders and rejection of defective supply batches

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ABSTRACT

We consider a single-echelon inventory installation under the classical EOQ (with backorders) paradigm to study the effects of supply quality on cost performance. Previous research on imperfect supply quality has focused on variants of the proportional (deterministic or random) yield problem, where supply batches are all accepted and then used, after screening any defects. In this paper we study an alternative setting where, entire supply batches may be defective (below quality standards) and therefore rejected on arrival. Note that such supply flow disruptions are analogous to those considered within the economic order quantity with disruptions (EOQD) area of research. We first present an exact model for the system expected cost and show its convexity. Optimal cost and respective values of the decision variables (i.e. planned order quantity and backorders) are then obtained in a closed form. Interestingly, under perfect supply quality, all expressions reduce to those of classical EOQ. Analytical and computational results reveal the serious impact of inferior supply quality on system cost as well as the sensitivity of the system parameters.

1. Introduction

One assumption of the classical economic order quantity (EOQ) model (Haris, 1913) is that the supply process is of perfect quality. Real supply processes, however, often fail to conform to specifications; so deliveries may deviate from agreed quantities or times (or both). In order to study the effects of various inferior quality manifestations, research aimed at relaxing the perfect supply assumption of the EOQ model (and optimizing the resulting relaxed models) is important and ongoing. Since this paper considers a particular type of inferior supply quality, we start by presenting some of the major results in two of the related research areas, often referred to as the *imperfect proportional yield* (IPY) and the *economic order quantity with disruptions* (EOQD) problems. In this context we focus on studies that only relax the assumption of the supply process, so assume deterministic demand rate as in the original EOQ model.

Under the IPY problem, it is assumed that a fraction, say Z, of the supplies received is defective (i.e. bellow quality standards). The parameter Z is considered to be either deterministic (fixed) or a random variable (which may be known, known but dependent on other unknown parameters, or unknown). Zipkin (2000) gives a good review of the various yield types and their effects. Focussing on EOQ-type models, Salameh and Jaber (2000) studied a joint lot sizing and inspection policy. They assumed 100% inspection of all supplies and that defective items are withdrawn (as one single batch) from inventory by the end of inspection/screening period. This batch is sold to customers in secondary markets. Directly related is the study by Cárdenas-Barrón (2000) which corrects an error in Salameh and Jaber (2000). The error found concerns the EOQ formula and does not really affect the underlying ideas. Goyal and Cárdenas-Barrón (2002) revisited the work of Salameh and Jaber (2000) and presented a simpler approach to determine the optimal lot size quantity. Chan et al. (2003) proposed an extension to the Salameh and Jaber (2000) model, assuming that defectives could be either rejected at a cost or sold at a lower price or reworked instantaneously. They assumed three timing options for selling these items: (i) sell them immediately as they are detected, at a discounted price, (ii) keep them in stock and sell them at the end of the production period, and (iii) keep them in stock and sell them at the end of the order cycle. Hayek and Salameh (2001) studied a model with imperfect quality items, where Z is a random variable, the production rate is finite and shortages are fully backordered. Papachristos and Konstantaras (2006) clarified a point related to the condition for preventing stock-outs and extended the model of Salameh and Jaber (2000) to the case in which withdrawing takes place at the end of the planning horizon. Eroglu and Ozdemir (2007) and Wee et al. (2007) independently extended the work of Salameh and Jaber (2000) to account for

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backorders. Chang and Ho (2009) revisited the work of Wee et al. (2007) and obtained a new expected net profit per unit time applying the renewal-reward theorem. Maddah and Jaber (2008) rectified a flaw in Salameh and Jaber's (2000) work related to the method of evaluating the expected profit per unit time. Cárdenas-Barrón (2009) developed an EPQ model for imperfect quality products with planned backorders where all the defective products are reworked in the same cycle. Also Cárdenas-Barrón et al. (2013) determined the optimal replenishment lot size and the optimal number of shipments for an EPQ model with imperfect quality products, rework and multiple shipments. Yassine et al. (2012) extended the EPQ models to deal with the issues of shipping the imperfect quality items through disaggregating (into smaller batches within a single production run) or consolidation (across multiple production runs). Khan et al. (2011a) investigated the effects of both imperfect quality items and inspection errors in the EOQ model [Hsu (2012) later corrected some minor formulae typos]. Hsu and Hsu (2013) studied an EOQ model with imperfect quality items, inspection errors, planned backorders and sales returns, and they obtained closed form expressions for the optimal order quantity and the maximum planned backorders. Vörös (2013) revisited the economic order quantity model when a lot may randomly contain defective items. He dropped the widely used assumption intended to avoid shortages in an inspection cycle and he proposed two simple lot sizing rules. Recently, Nasr et al. (2013) studied a variant of the EOQ model under random supply where every item received is of imperfect quality with the same probability. They explored the effect of quality correlation in the EOQ-type model with a random binomial supply process. For a comprehensive review of EOQ-type models with imperfect quality items, the reader is referred to Khan et al. (2011b).

The second research area of interest here on imperfect supply quality concerns the EOQD problem, where the supplier is not always able to deliver as the buyer might require. Research in this area was effectively instigated by Parlar and Berkin (1991). This studies an EOQ setting (with lost sales), where the supply process consists of alternating "wet" and "dry" periods of random length. During "wet" periods, supplies are available as required while the supply-flow is totally disrupted during "dry" periods. Considering "wet" and "dry" periods as two stationary and independent exponential random variables, they used the renewal theory analysis to model expected cost and derived some analytical properties for optimization.

Groenevelt et al. (1992) considered an EOQD model applied to manufacturing processes. In this context, disruption represented a machine breakdown and the authors investigated the effects of these disruptions on lot-sizing decisions. They showed that the optimal lot size subject to disruptions is always larger than these for the EOQ case. Berk and Arreola-Risa (1994) revisited the work of Parlar and Berkin (1991) by correcting some elements of the renewal analysis as well as the total cost. Snyder (2006) proposed a simple but effective approximation to the optimal order quantity for the EOQD (as formulated by Berk and Arreola-Risa (1994)). The author replaced an exponential term in the model with a constant one and this led to near optimal solution in closed form. Parlar and Berkin (1991) allowed a replenishment (in *wet* period) when the inventory level drops to zero. Parlar and Perry (1996) extended the previous EOQD model to allow for non-zero reorder points and also they assumed that all demand during (only) "dry" periods is backordered. Heimann and Waage (2007) developed a closed-form approximate solution for the EOQD problem proposed by Parlar and Perry (1996). A detailed review on inventory management with supply disruptions literature can be found in Atan and Snyder (2012).

Under this background, we consider the following imperfect yield problem within the EOQ (with backorders) paradigm. A single-item inventory installation faces a deterministic demand rate D. Supplies follow a fixed delivery schedule, with delivery interval T. The supply process is imperfect; so there is a finite probability p that any supply batch may be defective (i.e. bellow quality standards); we assume that defective delivery occurrences are independent of each other. To control the process, an "*all or none*" policy (see Parlar et al., 1995) is used. Consequently, all deliveries are inspected and, if found defective, they are rejected (so the respective supply batch quantity does not enter inventory). There are no emergency deliveries, so the batch quantity of any defective delivery is routinely added to the quantity of next supply delivery. Assuming finite holding and backorders cost and a fixed cost per delivery, we are seeking the delivery interval T (or equivalently the planned order quantity $Q=DT$) and the planned backorders J that minimize the long-run expected cost per unit time.

Although somewhat idealized, the simple system analyzed here has several realistic aspects and can be viewed in the context of both research areas presented above. First considering the IPY problem, the "*all or none*" policy is a limiting case of the proportional yield where the yield fraction Z is a binary random variable 0–1 (with probabilities p and $1-p$ respectively). Note that such a control scheme is real industrial use, often applied in connection with non-itemized bulk-purchased supplies. However, it is also used for itemized supplies under classical (sample-based) statistical acceptance control. In the EOQD setting, it corresponds to situation where the supplier (due to internal or external reasons) is unable to always deliver as planned. So by substituting defective deliveries and the quality control policy described above by "unrealized deliveries", the problem falls directly the EOQD setting. Interestingly, by understanding the random variables presenting the number of successive realized and unrealized deliveries as the "wet" and "dry" periods respectively, the system analyzed here is quite similar to that analyzed in Parlar and Perry (1996).

Based on this discussion, the contribution of this paper is summarized as follows: (i) we present an exact expected cost model of the system which is shown jointly convex in both decision variables (i.e. order quantity and planned backorders); (ii) we show that the model optimal solution and the respective optimizers can be obtained in closed-form (we know of no other EOQD-related model with such a property) and they all reduce to those of the classical EOQ model under perfect supply quality; (iii) we provide analytical results which demonstrate that always the optimal expected cost increases and the optimal order quantity decreases with decreasing supply quality (we know of no previous analytical results on the issue); (iv) we provide numerical results demonstrating the sensitivity of the optimal expected cost on changes in supply quality.

The remainder of this paper is organized as follows. Section 2 gives the model assumptions and notation used. Section 3 presents the exact expected cost model and the analysis leading to the optimal solution which is obtained in closed-form. Section 4 provides numerical results and a discussion of their implications in terms of system cost performance. Finally, Section 5 summarizes the findings and provides directions for future research.

2. Preliminaries

The major notation used and the operating assumptions underlying the model have as follows:

2.1. Notation

Q planned order quantity (in units) [decision variable]

J planned backorders (i.e. backorder level for a cycle of length $T'=T$) [decision variable]