Applied Mathematical Modelling 38 (2014) 1983–1999

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Inverse kinematics solutions for industrial robot manipulators with offset wrists

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ARTICLE INFO

Article history:
Received 31 January 2012
Received in revised form 31 July 2013
Accepted 8 October 2013
Available online 25 October 2013

Keywords:
Inverse kinematics
Dialectic elimination
Offset wrist
Analytical and numerical solutions

ABSTRACT

In this paper, the inverse kinematics solutions for 16 industrial 6-**D**egrees-of-**F**reedom (DOF) robot manipulators with offset wrists are solved analytically and numerically based on the existence of the closed form equations. A new numerical algorithm is proposed for the inverse kinematics of the robot manipulators that cannot be solved in closed form. In order to illustrate the performance of the **N**ew **I**nverse **K**inematics **A**lgorithm (NIKA), the simulation results attained from NIKA are compared with those obtained from well-known **N**ewton–**R**aphson **A**lgorithm (NRA). The inverse kinematics solutions of two robot manipulators with offset wrists are given as examples. In order to have a complete idea, the inverse kinematics solution techniques for 16 industrial robot manipulators are also summarized in a table.

1. Introduction

The solution of the inverse kinematics problem is computationally expensive and generally takes much time for real time control of robot manipulators. It is very important for robot design, trajectory planning and dynamic analysis of robot manipulators. There are mainly two types of inverse kinematics solutions techniques, namely analytical and numerical. Some problems encountered for robot manipulators are singularities (if determinant of the Jacobian matrix is zero, there are singular points and hence there is no solution) and nonlinearities that make the problem more difficult to solve. For a very small class of kinematically simple manipulators such as robot manipulators with Euler wrist (Fig. 1a) have been completely solved analytically [1].

In some industrial applications employed in welding, painting, cutting, material handling, machine tending and surgery etc., robot manipulators are forced to handle high payloads and to get long horizontal reach and appropriate angle. However, robot manipulators with Euler wrist whose three consecutive joint axes intersect a common point (Fig. 1a) may not meet these requirements. In order to meet these requirements, they can be equipped with offset wrists (Fig. 1b) whose three axes do not intersect in a common point. Some robot manipulators with offset wrist such as Panasonic VR-004GII, Kawasaki EE10, ABB IRB 2400, Kuka IR662, and Fanuc P145, etc. are very commonly used in industry. One way of solving inverse kinematics of these robot manipulators in closed form is to transform the kinematics equations into a 16 degree polynomial using the half-angle tangent of joint variables [2]. Unfortunately, obtaining the roots of 16 degree polynomial is very tedious and very time consuming. At the same time, analytical inverse kinematics solutions of some robot manipulators are difficult or impossible due to their complex kinematics structures. In the absence of closed form solution, numerical techniques are potential ways of solving the inverse kinematics problem. There have been developed two types of numerical methods to solve inverse

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0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.10.014

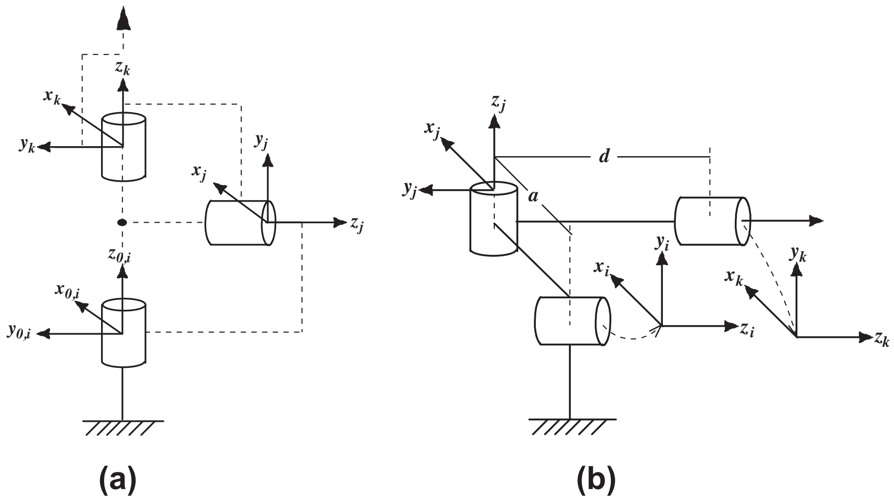

Fig. 1. The structures of (a) Euler wrist, (b) offset wrist.

kinematics problems. The first type employs Newton–Raphson method [3,4] for the nonlinear kinematics equations or predictor–corrector types algorithms [5,6] for differential kinematic equations. The main drawback of the first type results from obtaining no solutions in the singular points of Jacobian matrix (it fails to converge, when singularity exists) and requiring accurate initial solution vector. The second type uses the different gradient-based nonlinear algorithms (NP). This method converts the inverse kinematics problem into an equivalent minimization problem [5,7]. Since this method does not use inverse Jacobian matrix, it is more suitable than the first method. Because manipulator kinematics equations include high nonlinearity and the gradient vector of the objective function is computed numerically, computational cost increases very much.

Recently, Martins and Guenther [8] introduced a hierarchical analysis method for the robot inverse kinematics problem based on a graphical interpretation of the Jacobian matrix. Zhao et al. [9] developed a numerical algorithm for the inverse position analysis of all serial manipulators using inversion of Jacobian matrix. But inverting the Jacobian is not an easy task, especially when it is implemented online [10]. Moreover, singular points of Jacobian matrix are disastrous, especially in some robot configurations. Especially, robot manipulators with offset wrist have plenty of singular points in the workspace. Pashkevich [11] developed a real time inverse kinematics algorithm for robot manipulator with offset and reduced wrist. Although the algorithm does not use a Jacobian matrix, it fails to converge as the wrist configuration gets closer to the singularity (like $|q_5| < 0.5$ degrees). Also, there have been developed other methods based on heuristic search techniques such as neural networks [12–14], genetic algorithms [15,16] for the solution of inverse kinematics problem. However, from a real-time control point of view, they suffer from slow iterations. In this paper, instead of conventional techniques, a new iterative inverse kinematics algorithm (NIKA) is developed for the robot manipulators with offset wrist. Since the NIKA does not use a Jacobian matrix, it does not fail to converge, when singularity exists.

The inverse kinematics is one of the most complicated problems of robotics. In order to solve an inverse kinematics of a robot manipulator, one first should develop a mathematical model that produces suitable homogenous equations for the solution of the inverse kinematics. There may be suitable equations hidden somewhere, if one does not get suitable mathematical model that yields these hidden equations, the inverse kinematics cannot be solved in closed form although it is possible. Sometimes the inverse kinematics of very simple known classical serial mechanism like 'RRRRRR-planar robot manipulator' has not been obtained in closed form. The manipulator is very simple but there has not been solved in closed form until now. Therefore, although the robot manipulators considered in this paper are known classical serial mechanisms, some of these robot manipulators with offset wrists have not been solved in closed form. In this paper, some of these robot manipulators are solved in closed form by obtaining appropriate mathematical models and a numerical algorithm is developed for the remaining.

The paper is organized in the following manner. In Section II, the two-letter code description of robot configurations are presented. In Section III, the properties of homogenous transformation matrices are described and inverse kinematics solution techniques are explained. In Section IV, the inverse kinematics solutions of two robot manipulators are given as examples. Solution techniques of 16 robot manipulator with offset wrist are presented in a table. Finally, conclusions of this study are given in section V.

2. Two-letter code description of manipulator configurations

Huang and Milenkovic [17] used a two-letter code to classify robot configurations. The first letter characterizes the first joint and the first joint's relationship to the second joint. The second letter identifies the third joint and third joint's