Int. J. Production Economics 143 (2013) 162–170

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns

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ARTICLE INFO

Article history:
Received 9 January 2012
Accepted 26 December 2012
Available online 9 January 2013

Keywords:
EOQ
Imperfect quality
Misclassification errors
Inventory backordering

Abstract: In practice the items received in a lot may contain defective items, and the screening process to eliminate the defective items may involve both a Type I and a Type II error. Due to the defective items and the inspection errors, shortages may sometimes occur. In this paper, we develop an economic order quantity model with imperfect quality items, inspection errors, shortage backordering, and sales returns. A closed form solution is obtained for the optimal order size, the maximum shortage level, and the optimal order/reorder point. If shortages are not allowed, we adopt the order overlapping scheme proposed by Maddah et al. (2010) (Computers and Industrial Engineering. 58 (4), 691–695.) to develop an EOQ model to prevent shortages during the screening process. Numerical examples show the impacts of the defective probability, the inspection errors, and the holding and backordering costs on the optimal solution.

1. Introduction

One of the key assumptions of the basic economic order quantity (EOQ) model is that the items received are of perfect quality. However, product quality is not always perfect but is directly affected by the reliability of the production process used. Therefore, the process may deteriorate and produce defectives or poor quality items. Porteus (1986) introduced a model that shows a significant relationship between quality and lot size. He incorporated the effect of defective items into the basic EOQ model and introduced the option for investing in process quality improvement by means of reducing the probability that the process moves out of control. Lee and Rosenblatt (1987) addressed the problems of joint control of production cycles or manufacturing quantities, and maintenance by inspections. They solved the problem of simultaneous determination of economic manufacturing quantity (EMQ) and the inspection schedule by using an approximation method to the cost function. They also examined the cost penalties of using the classical EMQ and the effects of different parameters of the system on the magnitude of these penalties. Schwaller (1988) extended the EOQ model by adding the assumptions that defective items of a known proportion were present in incoming lots and that fixed and variable inspection costs were incurred in finding and removing these defective items. It was also assumed that the supplier reimbursed the buyer for any defective items found and removed. Cheng (1991) proposed an EOQ model with demand-dependent unit production cost and imperfect production processes. He formulated the inventory decision problem as a geometric program and solved it to obtain closed form optimal solutions. Ben-Daya and Hariga (2000) studied the effect of imperfect production processes on the economic lot sizing policy. Salameh and Jaber (2000) developed an economic order quantity model in which each ordered lot contains a random proportion of defective items. They assumed that received items are subject to 100% inspection with no inspection errors and that poor-quality items are kept in stock and sold as a single batch at a reduced price at the end of the 100% inspection process. Cárdenas-Barrón (2000) corrected an error in Salameh and Jaber's (2000) paper. Goyal and Cárdenas-Barrón (2002) presented a simple approach for determining the economic production quantity of an item with imperfect quality. They showed that near-optimal results are obtained by using the simple approach. The model suggested in their note is easier to implement. Chiu (2003) considered the economic production quantity model with the rework process of imperfect quality items. He assumed that not all of the defective items are repairable; a portion of them are scrap and are discarded. Chan et al. (2003) developed an economic production quantity (EPQ) model where the imperfect (not necessarily defective) items could be sold at a lower price and the defective items could be either reworked or rejected. Chang (2004) proposed two fuzzy models for an inventory problem with imperfect quality items. In the first model, the defective rate is represented by a fuzzy number, while the annual demand rate is treated as a fixed constant. In the second model, not only the defective rate but also the annual demand rate is represented by a fuzzy number.

Papachristos and Konstantaras (2006) discussed the issue of non- shortages in inventory models where the proportion of

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http://dx.doi.org/10.1016/j.ijpe.2012.12.025

defective items was a random variable. Speculating on the timing of withdrawing and selling the lot of imperfect items, they proposed an alternative to Salameh and Jaber's (2000) model where defectives were withdrawn at the end of the cycle rather than at the end of 100% inspection. Liao (2007) studied imperfect production processes that required maintenance. He considered two states, namely the in-control and the out-of-control state of the production process. The maintenance process would either worsen the production system or bring it to a perfect state. He showed that there exists a unique optimal number of imperfect maintenance processes (N) before the production system is brought to the perfect state. Wee et al. (2007) extended the approach by Salameh and Jaber (2000) to consider permissible shortage backordering and the effect of varying backordering cost values. Eroglu and Ozdemir (2007) also extended Salameh and Jaber's (2000) model by allowing shortages to be fully backordered. Maddah and Jaber (2008) corrected a flaw in Salameh and Jaber's (2000) model by using the renewal reward theory. They came up with simpler expressions for the expected profit and the order quantity. Cárdenas-Barrón (2009) developed an EPQ inventory model with planned backorders for determining the economic production quantity and the size of backorders for a single product, which was made in a single-stage manufacturing process that generated imperfect quality products and required that all defective products be reworked in the same cycle. Sana (2010) investigated an EPL (economic production lot size) model in an imperfect production system in which the production facility might shift from an 'in-control' state to an 'out-of-control' state at any random time. Stimulated by Papachristos and Konstantaras (2006), Maddah et al. (2010) revisited Salameh and Jaber's (2000) paper and proposed an order overlapping scheme that allowed the demand during the screening period of an order to be met from the inventory of the previous order. In other words, to prevent shortages during the screening process, an order is placed when the inventory level is just enough to cover the demand during the screening period. Then, the demand during the screening period of an order is met from the inventory of the previous order. Chang and Ho (2010) revisited the work of Wee et al. (2007) and adopted the suggestion of Maddah and Jaber (2008) to use the renewal reward theorem to derive closed-form solutions for the optimal lot size and the maximum shortage level without using differential calculus. Chung (2011) revisited the work of Cárdenas-Barrón (2009) and developed the sufficient and necessary condition for the existence of the solution satisfying the first derivative condition of the annual total relevant cost and presented a concrete solution procedure to find the optimal solution. Liao and Sheu (2011) described an integrated EPQ model that incorporated EPQ and maintenance programs. The optimal run time, required to minimize the total cost, was discussed. Hsu and Hsu (2012) pointed out a contradiction between Wee et al. (2007) and Chang and Ho's (2010) mathematical model and assumption, and developed a corrected model based on their assumption. For more recent works on inventory models with imperfect quality items, we refer the readers to Khan et al. (2011a).

Salameh and Jaber's (2000) model assumed that there was no human error in the screening process. Raouf et al. (1983) studied human error in inspection. They came up with one of the first inspection plans with misclassifications for multi-characteristic critical components. Duffuaa and Khan (2002) proposed a general inspection plan for quality assurance of critical multi-characteristic components. They extended Raouf et al. (1983) inspection plan for the case of six types of misclassification errors, where an inspector could classify an item to be good, rework or scrap. Duffuaa and Khan (2005) carried out a sensitivity analysis to investigate the statistical and economic impact of the several types of misclassification errors on the performance measures of the inspection plan. Yoo et al. (2009) proposed a profit-maximizing EPQ model that incorporated both imperfect production quality and two-way imperfect inspection (i.e., Type I and Type II inspection errors). They also considered rework and salvage in the disposing of screened and returned items, and solved the model optimally and presented numerical sensitivity analyses to provide important managerial insights into practices. Khan et al. (2011b) extended the work of Salameh and Jaber's (2000) model by assuming that the inspection process was not error-free. They used Raouf et al. (1983) approach to suggest that an inspector could incorrectly classify a non-defective item to be defective (a Type I error) or incorrectly classify a defective item to be non-defective (a Type II error). Hsu (2012a, 2012b) pointed out some errors in Khan et al. (2011b) paper. Hsu and Hsu (2013) developed two economic production quantity models with imperfect production processes, inspection errors, planned backorders, and sales returns.

Note that Khan et al. (2011b) assumed that shortage was not allowed. However, due to the defective items and inspection errors, shortages may sometimes occur. The purpose of this paper is to develop an optimal inventory model to determine when and how much to order for items with imperfect quality, inspection errors, shortage backordering, and sales returns. The rest of the paper is organized as follows. In the next section, the notation and model description are introduced. In Section 3, a mathematical model is developed for the case when shortages are allowed and are backordered. In Section 4, we develop the model for the case when shortages are not allowed. To prevent shortages during the screening process, we use the order overlapping scheme proposed by Maddah et al. (2010). Section 5 provides numerical examples to illustrate important aspects of the model. Finally, a general conclusion and future directions of the present study are provided in section six.

2. Notation and model description

We consider a buyer with a constant annual demand rate. The buyer places an order of size Q for each procurement cycle. It is assumed that the vendor's production processes are imperfect and may produce defective items. Once the buyer receives the lot, a 100% screening process is conducted. The screening process and demand proceed simultaneously. The screening process is also imperfect in that an inspector may incorrectly classify a non-defective item as defective, or a defective item as non-defective. We assume that when there is a shortage, all customers are willing to wait for the next delivery. The following notation is used:

D	the number of units demanded per year
Q	the order size
B	the maximum backordering quantity in units
OP	the order/reorder point
LT	the lead-time between the time an order is triggered and the time the order is received
$DDLT$	the demand during the lead-time, which is equal to LT times the demand per unit time
c	the unit variable cost
K	the ordering cost per order
s	the unit selling price of a non-defective item
v	the unit selling price of a defective item
x	the screening rate
d	the unit screening cost
h	the holding cost per unit per unit time
b	the backordering cost per unit per unit time
T	the cycle length
p	the probability that an item is defective