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The (σ, S) policy for uncertain multi-product newsboy problem

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ABSTRACT

This paper derives an optimal (σ, S) policy for uncertain multi-product newsboy problem. Demands for the products are estimated by experts and assumed to be independent uncertain variables. Uncertainty theory, which is a new mathematical tool to deal with human uncertainty, is employed to model demand distributions. A fixed setup cost and a linear ordering cost are incurred if products are ordered. Setup cost is variant and depends on whether a joint order or an individual order is placed. A methodology is proposed for determining the optimal (σ, S) policy. Finally, a two-product example is provided to show how to design an optimal (σ, S) policy in realistic situation.

1. Introduction

The replenishment (σ, S) policy for multi-product stochastic inventory system has been widely studied. Under (σ, S) policy, if the inventory level $\boldsymbol{x}$ whose elements are initial inventory level x_i of each product i is in a reorder set σ, then the inventory level is brought up to S, otherwise nothing is ordered. Such a policy reduces to the well-known (s, S) policy in the single product case. The fundamental works on optimal inventory ordering policy can date back to Arrow, Harris, and Marschak (1951). In addition, due to the practical and theoretical importance of multi-product newsboy model, Hadley and Whitin (1963) presented their seminal work on multi-product newsboy model.

One of the important factors affecting order policy is the nature of the demand. In conventional newsboy models, demands are described by probability distributions. Scarf (1958) first investigated a distribution free newsboy model in which only the mean μ and the variance σ^2 are known. Some other authors intended to use the normal distribution to estimate demand distribution. However, normal demand distribution has a disadvantage. Demand value could be negative. Hence, Veinott and Wagner (1965) assumed that the demands for products are non-negative, discrete random variables. Some other researchers described the demand by continuous random variable. Mahoney and Sivazlian (1980) assumed that demands for products have Erlang distributions. Lau and Lau (1995) assumed that demand for each product is exponentially distributed. Dominey and Hill (2004) assumed that demand follows a compound Poisson distribution. Berk, Gürler, and Levine (2007) adopted Bayesian approach to estimate gamma demands. And, Abdel-Malek and Otegbeye (2013) applied moments approximation technique to simulate stochastic distributions.

Several scholars proposed extensions to the newsboy problem where the demand does not satisfy the classical probability distribution assumptions. Gerchak and Mossman (1992) applied mean preserving transformation $X_\alpha = \alpha x + (1-\alpha)\mu, \alpha \geqslant 0$ (x is a random variable with mean μ) to model changes in demand randomness. Vairaktarakis (2000) described demand uncertainty using interval and discrete scenarios, and proposed deterministic optimization models. Huang, Zhou, and Zhao (2011) considered demand consists of the initial demand and the demand derived from other products' unmet demand.

However, in most cases, the distributional information of the demand is very limited. Sometimes all available is just belief degree of distribution given by expert. Because human beings usually overweight unlikely events (Kahneman & Tversky, 1979), variance of belief degree may be much larger than real frequency. In this situation, if we deal with the belief degree using probability theory, some counterintuitive results will be obtained (Liu, 2012). In order to deal with belief degree, Liu (2007) founded the uncertainty theory, and refined it (Liu, 2009a). From then on, the uncertainty theory has been a branch of mathematics to deal with human uncertainty and applied in many fields. In order to model optimization problems with uncertain parameters, Liu (2009b) proposed uncertain programming. In addition, Liu and Ha (2010) proved a formula for calculating the expected values of monotone functions of uncertain variables. Gao (2011) investigated the α-shortest path and the shortest path problem in uncertain networks. Recently, uncertainty theory has been extended to the fields of uncertain stationary independent increment process (Liu, 2008), uncertain inference (Gao, Gao, & Ralescu, 2010), uncertain logic (Chen &

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Ralescu, 2011), uncertain alternating renewal process (Yao & Li, 2012), and entropy (Chen, Kar, & Ralescu, 2012).

A few authors suggested that uncertainty theory provides the appropriate base to treat demand uncertainty. Gao (2010) proposed the optimal (s, S) policy for uncertain newsboy. Qin and Kar (2013) assumed that the uncertainty appears in the market demand for each product and derived the optimum order quantity formula to maximize the expected profit. But, they considered only one product newsboy problem.

The uncertain multi-product newsboy problem described in this paper could be seen as an expansion of classical newsboy problem. There are a large number of literatures on newsboy problem with random demand. However, in some cases, we cannot obtain the probability distribution of the demand due to lack of enough samples. For instance, the probability distribution of the demand of newspaper may not be got because of the influence of unexpected political events. In order to deal with demand in a realistic way, we adopt uncertainty theory to handle indeterminacy information and assume that demand is an uncertain variable. On the other hand, though a few scholars have applied uncertainty theory to newsboy problem, they all consider only one product newsboy problem. This paper expands their models to uncertain multi-product newsboy problem with a fixed joint setup cost. The main contribution of this paper is to develop an optimal (σ, S) policy so as to maximize the uncertain expected profit of newsboy.

The remainder of this paper is organized as follows. In Section 2, some preliminary concepts of uncertainty theory are introduced. In Section 3, a single-period multi-product uncertain inventory model is described in detail. In Section 4, a methodology to derive optimal (σ, S) policy is proposed. In Section 5, a case study is presented to illustrate how to design an optimal (σ, S) policy. Finally, in Section 6, concluding summary is provided.

2. Preliminaries

Let Γ be a nonempty set, and let $\mathcal{L}$ be a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is assigned a number $\mathcal{M}\{\Lambda\} \in [0, 1]$. In order to ensure that the $\mathcal{M}\{\Lambda\}$ has certain mathematical properties, Liu (2007, 2009a) presented four axioms: (1) normality axiom, (2) duality axiom, (3) subadditivity axiom, and (4) product axiom.

Definition 1 (Liu, 2007). Let Γ be a nonempty set, $\mathcal{L}$ a σ-algebra over Γ, and $\mathcal{M}$ an uncertain measure. Then the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

Definition 2 (Liu, 2007). An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\},$$

is an event.

Definition 3 (Liu, 2009a). The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}\{\xi_i \in B_i\}\right\} = \min_{1 \leqslant i \leqslant n} \mathcal{M}\{\xi_i \in B_i\},$$

for any Borel sets $B_1, B_2, \ldots, B_n$ of real numbers.

The uncertainty distribution of an uncertain variable ξ is defined by $\Phi(x) = \mathcal{M}\{\xi \leqslant x\}$ for any real number x. For example, the linear uncertain variable $\xi \sim \mathcal{L}(a, b)$ has an uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leqslant a \\ (x-a)/(b-a), & \text{if } a \leqslant x \leqslant b \\ 1, & \text{if } b \leqslant x. \end{cases}$$

Definition 4 (Liu, 2010). Let ξ be an uncertain variable with regular uncertainty distribution Φ. Then the inverse function Φ^{-1} is called the inverse uncertainty distribution of ξ.

Definition 5 (Liu, 2010). An uncertainty distribution Φ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$.

Example 1. The inverse uncertainty distribution of linear uncertain variable $\mathcal{L}(a, b)$ is

$$\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b.$$

Obviously, linear uncertain variable has a regular uncertainty distribution. In practical application, we usually assume that all uncertainty distribution is regular. Otherwise, some perturbations can be used to get a regular one.

Theorem 1 (Liu, 2010). *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If f is a strictly increasing function, then*

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_n),$$

is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)\right).$$

Theorem 2 (Liu, 2007). *Let ξ be an uncertain variable with uncertainty distributions Φ. If the expected value exists, then*

$$E[\xi] = \int_0^{+\infty}(1-\Phi(x))\mathrm{d}x - \int_{-\infty}^{0}\Phi(x)\mathrm{d}x.$$

Definition 6 (Liu, 2010). Let ξ be an uncertain variable with regular uncertainty distribution Φ. If the expected value exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)\mathrm{d}\alpha.$$

Assume that demands are uncertain variables. We will investigate an expected value model for uncertain multi-product newsboy problem in Section 3.

3. Model description

We consider a single-period multi-product inventory model operating under a (σ, S) policy. The demand for each product is described by the independent uncertain variable with uncertainty distribution function. A decision for replenishing stock is possible at the beginning of the period through individual or joint orders. We assume that delivery of orders is instantaneous and the ordering cost consists of a linear ordering cost and a fixed setup cost. The objective is to develop the optimal (σ, S) policy for uncertain multi-product newsboy problem.

In order to formulate the expected profit for uncertain multi-product newsboy problem, we list the notations for product i, $i = 1, 2, \ldots, n$ as below.