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Determining the optimal allocation of parameters for multivariate measurement system analysis

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A B S T R A C T

Measurement system analysis (MSA) plays an important role in helping organizations to improve their product quality. Usually, a gauge repeatability and reproducibility (GRR) study needs to be conducted prior to the process capability analysis for assessing the adequacy of gauge variation. In performing multivariate MSA for multivariate measurement systems, the existing multivariate precision-to-tolerance (P/T) ratio does not take the correlation coefficients among tolerances into account. Moreover, the optimal allocation of several parameters such as the number of quality characteristics (v), sample size of parts (p), number of operators (o) and replicate measurements (r) have not been considered in the multivariate GRR study either. As the total number of measurements (n) increase, the estimated total variation becomes more precise, but the related inspection time and costs will be increased as well. Striking the right balance between the precision of measurement system while still maintaining cost-effectiveness in determining the optimal allocation of *opr* parameters for correlated quality characteristics becomes an important issue in practical applications.

In this paper, a revised precision-to-tolerance (P/T) ratio for multivariate MSA with correlated quality characteristics is proposed. The simulation results show that our revised P/T ratio outperforms the existing ones in terms of MSE and MAPE. We have also found that two parameters (number of parts p and a total number of measurements n) significantly affect the expected length of confidence interval of P/T_R. A reference table with the optimal allocation of *por* parameters is constructed accordingly if the inspection cost is limited. Finally, a numerical example with a step-by-step procedure for conducting a MGRR study is given to illustrate the appropriateness of our proposed P/T ratio. Hopefully, it can be served as a useful guideline for quality practitioners when performing a multivariate measurement system analysis in industries.

1. Introduction

Measurement system analysis (MSA) plays an important role in helping organizations to improve their product quality. Generally speaking, the gauge repeatability and reproducibility (GRR) study is performed according to the MSA handbook stated in QS9000 standards. Usually, a GRR study needs to be conducted prior to the process capability analysis for assessing the adequacy of gauge variation. Good quality products can only be achieved through an adequate measurement system. Hence, finding ways to ensure the quality of a measurement system becomes an important task for quality practitioners. Moreover, in performing the GRR study, most industries today are using the approval criteria of precision to tolerance (P/T) ratio as stipulated in QS9000. Traditional MSA only considers a single quality characteristic. With the advent of modern technology, industrial products have become very sophisticated with more than one quality characteristic and there is high correlation among them, which can be found in solder paste stencil printing process (Pan & Lee, 2010). In performing the multivariate MSA for correlated quality characteristics, quality practitioners are also expected to determine the optimal allocation of sample size of parts (p), number of operators (o) and repeated measurements (r) for economic reasons. As the total number of measurements (n) increase, the estimated total variation becomes more precise, but the related inspection time and costs will be increased as well. Striking the right balance between the precision of measurement system while still maintaining cost-effectiveness in determining the optimal allocation of *por* parameters when conducting

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multivariate GRR study for correlated quality characteristics is the main purpose of this research. However, the adequacy of the current *por* selection method for conducting multivariate GRR is questionable. Thus, it becomes necessary to provide a guideline of *por* selection for quality practitioners when conducting multivariate GRR study for correlated quality characteristics.

Recently, principle component analysis (PCA) related methods are proposed by He, Wang, and Cook (2011), Osma (2011), Peruchi, Balestrassi, de Paiva, Ferreira, and de Santana Carmelossi (2013) and Wang (2013) for analyzing gauge variation. He et al. (2011) proposed a modified principal component analysis (PCA) method to transform multivariate measurement data with dependent variables into those with independent principal components. Peruchi et al. (2013) and Peruchi, Paiva, Balestrassi, Ferreira, and Sawhney (2014) proposed a weighted principal component (WPA) method for multivariate analysis of a measurement system, which can be considered as a special PCA approach. Wang (2013) applied PCA to perform assessment for multivariate GRR. Although He's and the other existing PCA methods can be served as an alternative approach to multivariate MSA, these methods cannot provide/remove the causes of production problems as the original "unit of measure" for the quality characteristics will be disappear after performing PCA. On the other hand, multivariate analysis of variance (MANOVA) proposed by Majeske (2008) does not have such problems. But, Majeske (2008) did not take the correlation coefficients among tolerances into account. Hence, in this paper, a revised precision-to-tolerance (P/T) ratio is proposed to fill the research gap in determining the optimal allocation of *opr* parameters when performing multivariate MSA for correlated quality characteristics.

2. Literature review

2.1. The gauge repeatability and reproducibility (GRR)

The purpose of MSA is to qualify a measurement system for use by quantifying its accuracy, precision, and stability. Usually, gauge repeatability and reproducibility (GRR) study for assessing the adequacy of gauge precision needs to be conducted prior to the process capability analysis. According to AIAG (2010), the variance of the measured values can be uniquely partitioned into two components:

$$\sigma^2_{obs} = \sigma^2_{part} + \sigma^2_{gauge} \tag{1}$$

where σ^2_{part} is the component of variance due to the product and σ^2_{gauge} is the component of variance due to measurement. Moreover, the variability of measurement can be further defined as:

$$\sigma^2_{gauge} = \sigma^2_{repeatability} + \sigma^2_{reproducibility} \tag{2}$$

where $\sigma^2_{repeatability}$ represents repeatability and $\sigma^2_{reproducibility}$ represents reproducibility. The information obtained from GRR study can be used to quantify the variations, which provides useful guidance for improving the accuracy and precision of a measurement system.

2.2. ANOVA analysis for univariate GRR

To estimate the potential sources of measurement variation, two methods commonly used in GRR study are: (1) $\overline{X}$–R chart method and (2) analysis of variance (ANOVA) method. Because the $\overline{X}$–R method cannot calculate the variance of operator-by-part interaction, Montgomery and Runger (1993) proposed the ANOVA method which uses a two-way random-effects ANOVA model with the interaction term. The random-effects model is defined as:

$$y_{ijk} = \mu + P_i + O_j + (PO)_{ij} + \varepsilon_{ijk} \quad \begin{cases} i = 1,\ldots,p \\ j = 1,\ldots,o \\ k = 1,\ldots,r \end{cases} \tag{3}$$

where y_{ijk} is the individual value of measurement, μ is the measurement mean (total mean), P_i is the effect of product, O_j is the effect of operator, $(PO)_{ij}$ is the effect of interaction between product and operator, ε_{ijk} is the effect of replicate measurements, p is the sample size of parts, o is the number of operators, and r is the number of replications. The effects of $P_i, O_j, (PO)_{ij}$ and ε_{ijk} are assumed to be normally distributed with means of zero and variances $\sigma^2_p, \sigma^2_o, \sigma^2_{po}$ and σ^2_ε, respectively. This is, $P_i \sim N(0,\sigma^2_p), O_j \sim N(0,\sigma^2_o), (PO)_{ij} \sim N(0,\sigma^2_{po})$ and $\varepsilon_{ijk} \sim N(0,\sigma^2_\varepsilon)$. By using expected mean square (EMS), one can obtain the estimated values of sources of variation, which are listed as follows:

$$\hat{\sigma}^2_{repeatability} = \hat{\sigma}^2_\varepsilon = MS_E \tag{4}$$

$$\hat{\sigma}^2_{reproducibility} = \hat{\sigma}^2_o + \hat{\sigma}^2_{po} = \frac{(p-1)MS_{po} + MS_o - pMS_E}{pr} \tag{5}$$

Then the variability of gauge can be calculated through the following:

$$\hat{\sigma}^2_{gauge} = \hat{\sigma}^2_{repeatability} + \hat{\sigma}^2_{reproducibility} = \frac{(p-1)MS_{po} + MS_o + p(r-1)MS_E}{pr} \tag{6}$$

If interaction between product and operator does not exist, then the repeatability, reproducibility, and variability of gauge can be calculated as:

$$\hat{\sigma}^2_{repeatability} = \hat{\sigma}^2_\varepsilon = MS_E \tag{7}$$

$$\hat{\sigma}^2_{reproducibility} = \hat{\sigma}^2_o = \frac{MS_o - MS_E}{pr} \tag{8}$$

$$\hat{\sigma}^2_{gauge} = \hat{\sigma}^2_{repeatability} + \hat{\sigma}^2_{reproducibility} = \frac{MS_o + (pr-1)MS_E}{pr} \tag{9}$$

2.3. The approval criteria for univariate GRR

To determine whether the precision of a measurement system is adequate or not, three commonly used criteria are listed below:

(1) Precision-to-tolerance ratio (P/T ratio)
According to AIAG (2010), P/T ratio is defined as:

$$\mathrm{P/T} = \frac{6\sigma_{gauge}}{TOL} \times 100\% \tag{10}$$

where TOL is the specification tolerance and σ_{gauge} is the standard deviation of the measurement system. The P/T ratio represents the percent of the specification tolerance taken up by measurement error. 6 standard deviation, $6\sigma_{gauge}$, accounts for 99.73% of measurement system variation based on normal distribution for underlying gauge error. Moreover, AIAG (2010) suggested the following guidelines for accepting gauge precision of a measurement system as listed in Table 1.

Table 1
GRR approval criterion suggested by AIAG.

P/T ratio	Decision
$\mathrm{P/T} \leqslant 10\%$	The measurement system is considered to be acceptable
$10\% < \mathrm{P/T} \leqslant 30\%$	The measurement system is considered to be marginally acceptable (may be acceptable for some applications)
$\mathrm{P/T} > 30\%$	The measurement system is unacceptable