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The use of OWA operator weights for cross-efficiency aggregation

Ying-Ming Wang [a,*], Kwai-Sang Chin [b]

[a] Institute of Decision Science, School of Public Administration, Fuzhou University, Fuzhou 350108, PR China
[b] Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong

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ABSTRACT

Cross-efficiency evaluation is an effective way of ranking decision making units (DMUs) in data envelopment analysis (DEA). Existing approaches for cross-efficiency evaluation are mainly focused on the calculation of cross-efficiency matrix, but pay little attention to the aggregation of the efficiencies in the cross-efficiency matrix. The most widely used approach is to aggregate the efficiencies in each row or column in the cross-efficiency matrix with equal weights into an average cross-efficiency score for each DMU and view it as the overall performance measurement of the DMU. This paper focuses on the aggregation process of the efficiencies in the cross-efficiency matrix and proposes the use of ordered weighted averaging (OWA) operator weights for cross-efficiency aggregation. The use of OWA operator weights for cross-efficiency aggregation allows the decision maker (DM)'s optimism level towards the best relative efficiencies, characterized by an orness degree, to be taken into consideration in the final overall efficiency assessment and particularly in the selection of the best DMU.

1. Introduction

Cross-efficiency evaluation, proposed by Sexton at al. [1], is an effective way of ranking decision making units (DMUs). It allows the overall efficiencies of the DMUs to be evaluated through both self- and peer-evaluations. The self-evaluation allows the efficiencies of the DMUs to be evaluated with the most favorable weights so that each of them can achieve its best possible relative efficiency, whereas the peer-evaluation requires the efficiency of each DMU to be evaluated with the weights determined by the other DMUs. The self-evaluated efficiency and peer-evaluated efficiencies of each DMU are then averaged as the overall efficiency of the DMU. Since its remarkable discrimination power, the cross-efficiency evaluation has found significant number of applications in a wide variety of areas such as the selections of R&D projects [2,3], flexible manufacturing systems (FMSs) [4], industrial robots [5] and computer numerical control (CNC) machines [6], preference voting and project ranking [7,8], cellular layouts evaluation [9], overall efficient electricity distribution sectors identification [10], the best labor assignment determination in cellular manufacturing system (CMS) [11], economic-environmental performance assessment [12,13], Olympic ranking and benchmarking [14–16], the efficiency evaluation of information sharing in supply chains [17], etc.

Besides a large number of applications, theoretical research has also been extensively conducted on the cross-efficiency evaluation. For example, Doyle and Green [18,19] examined in detail the idea of cross-efficiency, both mathematically and intuitively, and presented mathematical formulations of intuitive meanings for possible implementations of aggressive and benevolent cross-efficiencies. Liang et al. [20] proposed the concept of game cross-efficiency and developed a game cross-efficiency model which treats each DMU as a player that seeks to maximize its own efficiency under the condition that the cross-efficiency of each of the other DMUs does not deteriorate. The game cross-efficiency model was later extended to variable returns to scale (VRS) by Wu et al. [16]. Interested readers may refer to Anderson et al. [21] for the fixed weighting nature of the cross-efficiency evaluation in the case of single input and multiple outputs, Sun and Lu [22] for cross-efficiency profiling (CEP) analysis, Bao et al. [23] for an alternative interpretation to the cross-efficiency evaluation from the viewpoint of slack analysis in DEA, Liang et al. [24] for alternative secondary goals for the cross-efficiency evaluation, Wang and Chin [25,26] for more alternative DEA models for cross-efficiency and cross-weight evaluations, Wu [27] for the construction of fuzzy preference relation using cross-efficiencies, Wu et al. [28] for considering the performance rankings of the DMUs as a secondary goal, and Ramón et al. [29] for the choice of weights profiles in cross-efficiency evaluations.

Existing researches on the cross-efficiency evaluation are mainly focused on either its applications or the calculation of cross-efficiency matrix. Little attention has been paid to the aggregation process of cross-efficiencies. The most extensively used approach is to aggregate cross-efficiencies with equal weights. Our literature review reveals that only Wu et al. [30,31] determined ultimate cross-efficiency by weighting n cross-efficiency scores rather than simply averaging them. The weights

* Corresponding author. Tel.:+86 591 22866681; fax: +86 591 22866677.
E-mail addresses: msymwang@hotmail.com, ymwang@fzu.edu.cn (Y.-M. Wang).

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they utilized for aggregation were determined in terms of the nucleolus solution and the Shapley value in cooperative game, respectively.

In our view, the use of equal weights for cross-efficiency aggregation has a significant drawback. That is self-evaluated efficiencies are much less weighted than peer-evaluated efficiencies. This is because each DMU has only one self-evaluated efficiency value, but multiple peer-evaluated efficiency values. When they are simply averaged together, the weight assigned to the self-evaluated efficiency is only $1/n$ if there are n DMUs to be evaluated, whereas the remaining weights $(n-1)/n$ are all given to those peer-evaluated efficiencies. Quiet obviously, self-evaluated efficiencies fail to play a sufficient role in the final overall assessment and ranking. More importantly, the use of equal weights for aggregation has no way to take into consideration the decision maker (DM)'s subjective preferences on the best relative efficiencies in the final overall assessment and ranking. Besides, the assignment of a fixed but different weight to each DMU as done in Wu et al. [30,31] suffers from another significant drawback. That is the self-evaluated efficiency of each DMU plays a distinct role in the final overall assessment and ranking due to the fact that the self-evaluated efficiencies lie on the leading diagonal of the cross-efficiency matrix and the weights assigned to them differ from one DMU to another.

To overcome these drawbacks, we propose the use of ordered weighted averaging (OWA) operator weights for aggregating cross-efficiencies. The use of OWA operator weights for the cross-efficiency aggregation allows the weights to be reasonably allocated between self- and peer-evaluated efficiencies in terms of the DM's optimism level, characterized by an orness degree. By adjusting the DM's optimism level, self-evaluated efficiencies can play a desirable role in the final overall efficiency assessment or ranking of the DMUs.

The remainder of the paper is organized as follows: Section 2 briefly reviews OWA operators and their weight determination methods. Section 3 introduces the cross-efficiency evaluation in DEA. Section 4 proposes the use of OWA operator weights for cross-efficiency aggregation and discusses various possible preferences of the DM. Section 5 provides an illustrative example to demonstrate the potential applications of OWA operator weights in cross-efficiency aggregation. Section 6 concludes the paper.

2. OWA operators and their weight determination methods

An OWA operator of dimension n is a mapping $F:\Re^n \to \Re$ with an associated weight vector $W=(w_1,\ldots,w_n)^T$ such that

$$w_1+\cdots+w_n=1,\; 0\le w_i\le 1,\quad i=1,\ldots,n,$$

and

$$F(a_1,\ldots,a_n)=\sum_{i=1}^{n} w_i b_i,$$

where b_i is the ith largest of $a_1,\ldots,a_n$.

OWA operators, introduced by Yager [32], provide a unified framework for decision making under uncertainty, where different decision criteria such as maximax (optimistic), maximin (pessimistic), equally likely (Laplace) and Hurwicz criteria are characterized by different OWA operator weights. The following OWA operator weights lead to the well-known decision criteria for decision making under uncertainty:

(i) $W=(1,0,\ldots,0)^T$: In this case $F(a_1,\ldots,a_n)=\mathrm{Max}_i(a_i)$, which is the purely optimistic decision making (maximax criterion).

(ii) $W=(0,\ldots,0,1)^T$: In this case $F(a_1,\ldots,a_n)=\mathrm{Min}_i(a_i)$, which is the purely pessimistic decision making (maximin criterion).

(iii) $W=(1/n,\ldots,1/n)^T$: In this case $F(a_1,\ldots,a_n)=(1/n)\sum_{i=1}^{n} a_i$, which is the equally likely decision making (Laplace decision criterion).

(iv) $W=(\alpha,0,\ldots,0,1-\alpha)^T$: $F(a_1,\ldots,a_n)=\alpha\,\mathrm{Max}_i(a_i)+(1-\alpha)\mathrm{Min}_i(a_i)$, which is the Hurwicz decision criterion.

For different weight selections, they are distinguished by the following orness degree [32]:

$$\mathrm{orness}(W)=\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i,$$

which lies in the unit interval [0,1] and measures the degree to which the aggregation is like an *or* operation. The orness degree can be regarded as a measure of the optimism level of the DM.

To apply OWA operators for decision making, it is essential to determine the weights of OWA operators. The following models (1) and (2) are two important approaches for determining OWA operator weights under a given orness degree:

$$\begin{aligned}
&\text{Maximize}\;\; \mathrm{Disp}(W)=-\sum_{i=1}^{n} w_i \ln w_i,\\
&\text{Subject to}\;\; \mathrm{orness}(W)=\alpha=\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i,\;\; 0\le\alpha\le 1,\\
&\qquad \sum_{i=1}^{n} w_i=1,\\
&\qquad w_i\ge 0,\;\; i=1,\ldots,n,
\end{aligned}\tag{1}$$

and

$$\begin{aligned}
&\text{Minimize}\;\; \delta\\
&\text{Subject to}\;\; \mathit{orness}(W)=\alpha=\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i,\;\; 0\le\alpha\le 1,\\
&\qquad \sum_{i=1}^{n} w_i=1,\\
&\qquad w_i-w_{i+1}-\delta\le 0,\;\; i=1,\ldots,n-1,\\
&\qquad w_i-w_{i+1}+\delta\ge 0,\;\; i=1,\ldots,n-1,\\
&\qquad w_i\ge 0,\;\; i=1,\ldots,n.
\end{aligned}\tag{2}$$

Model (1), suggested by O'Hagan [33], maximizes the entropy of weight distribution and is thus referred to as the maximum entropy method, whereas model (2) that was proposed by Wang and Parkan [34] minimizes the maximum disparity between two adjacent weights and is thus called the minimax disparity approach.

The OWA operator weights determined by the above models have the following characteristics:

- The weights are ordered. That is $w_1\ge w_2\ge\cdots\ge w_n\ge 0$ if the orness degree $\alpha>0.5$ and $0\le w_1\le w_2\le\cdots\le w_n$ if $\alpha\le 0.5$.
- The weights have nothing to do with the magnitudes of the aggregates $b_1\sim b_n$, but depend upon their ranking orders and the DM's optimism level (orness degree).
- $w_1=1$ and $w_j=0$ $(j\ne 1)$ if $\alpha=1$, which means that the DM is purely optimistic and considers only the biggest value $b_1=\max_i(a_i)$ in decision analysis.
- $w_n=1$ and $w_j=0$ $(j\ne n)$ if $\alpha=0$, which represents that the DM is purely pessimistic and is only concerned with the most conservative value $b_n=\min_i(a_i)$ when making decision.
- $w_1=\cdots=w_n=(1/n)$ if $\alpha=0.5$, which stands for that the DM is neutral and makes use of all the aggregates $b_1\sim b_n$ equally in decision making.
- $w_1,\ldots,w_n$ determined by model (1) vary in the form of geometric progression, i.e. $w_{i+1}/w_i\equiv q$ for $i=1,\ldots,n-1$, where $q>0$, while $w_1,\ldots,w_n$ determined by model (2) vary in the form of