Journal of Banking & Finance 60 (2015) 127–137

Contents lists available at ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Portfolio optimisation with jumps: Illustration with a pension accumulation scheme

Olivier Le Courtois [a], Francesco Menoncin [b,*]

[a] EMLyon Business School, Avenue Guy de Collongue, 23 – 69130 Ecully, France
[b] University of Brescia, Via S. Faustino, 74/B – 25122 Brescia, Italy

ARTICLE INFO

Article history:
Received 8 November 2014
Accepted 3 August 2015
Available online 7 August 2015

JEL classification:
G13
G22

Keywords:
Optimal portfolio
Lévy process
Stochastic exponential
Pension fund

ABSTRACT

In this paper, we address portfolio optimisation when stock prices follow general Lévy processes in the context of a pension accumulation scheme. The optimal portfolio weights are obtained in quasi-closed form and the optimal consumption in closed form. To solve the optimisation problem, we show how to switch back and forth between the stochastic differential and standard exponentials of the Lévy processes. We apply this procedure to both the Variance Gamma process and a Lévy process whose arrival rate of jumps exponentially decreases with size. We show through a numerical example that when jumps, and therefore asymmetry and leptokurtosis, are suitably taken into account, then the optimal portfolio share of the risky asset is around half that obtained in the Gaussian framework.

1. Introduction

The traditional approach to portfolio and consumption optimisation is based on Brownian motion. However, this approach became outdated with the last financial crisis. In this paper, we use Lévy processes to take into account the empirical properties of asset returns, such as asymmetry and leptokurtosis, and check how they affect the optimal portfolio and consumption. We rely on a pension scheme because non-self-financing and long-term strategies have an increased role in modern economies. The paper develops a complete framework enabling us to compute optimal portfolio weights and consumption for arbitrary choices of Lévy processes used to model asset return dynamics.

The seminal contributions to portfolio optimisation with jumps are those of Aase (1984), Aase and Øksendal (1988), and Jeanblanc and Pontier (1990). Benth et al. (2001) numerically compute the optimal portfolio allocations when the asset returns follow Lévy processes with Normal Inverse Gaussian marginal distributions. In two companion papers, Benth et al. (2001), Benth et al. (2001) use a viscosity approach to tackle portfolio optimisation in the presence of jumps. Liu et al. (2003) develop a framework where Poisson jumps impact the dynamics of stocks and stock volatilities. Asset allocation problems in which the asset return dynamics are modelled with Lévy processes can also be found in Kallsen (2000), Framstad et al. (2001), Emmer and Klüppelberg (2004), Cvitanic et al. (2008), and Aït-Sahalia et al. (2009). Aït-Sahalia and Hurd (2012) have recently studied jumps and markets with contagion. The portfolio optimisation in a static setting with non-Gaussian laws has been studied by Han and Rachev (2000), Ortobelli et al. (2003), and de Athayde and Flôres (2004). Kennedy et al. (2009) study dynamic hedging with jumps.

In this work, we solve the optimal consumption and portfolio problem of an agent with Constant Relative Risk Aversion (CRRA) preferences who maximises the inter-temporal expected utility of consumption over his/her life-cycle. The agent can invest in a riskless asset and a risky asset whose return is modelled by a general Lévy process. The agent gets access to a constant pension flow starting at a given future date by accumulating constant contributions into a fund. The agent's lifetime is accordingly split into two periods: (i) the so-called "accumulation phase", when (constant) contributions are withdrawn from the agent's wealth and put into the fund and (ii) the so-called "distribution phase", when pensions are received by the agent and can increase his/her consumption.

This framework is akin to the 401(k) defined contribution pension plan in the United States where withdrawals from the fund are free of penalties only after a given age (59 and a half).

Defined contribution (DC) schemes are widespread among European pension funds. The financial literature concentrates its

* Corresponding author. Tel.: +39 030 2988896; fax: +39 030 2988837.
E-mail address: francesco.menoncin@unibs.it (F. Menoncin).

http://dx.doi.org/10.1016/j.jbankfin.2015.08.001
0378-4266/

efforts mainly on these schemes (see Haberman and Sung, 1994; Haberman and Vigna, 2001; Battocchio and Menoncin, 2004; Hainaut and Deelstra, 2011), while a few papers deal with the case of defined benefit (DB) schemes (Josa-Fombellida and Rincón-Zapatero, 2010). To our knowledge, only Josa-Fombellida and Rincón-Zapatero (2012) tackle the problem of a pension scheme with both contributions and pension income driven by jumps. Here, we concentrate on the case of a consumer/investor who deals with both constant contributions and pension income while maximising the expected utility of his/her inter-temporal consumption. Considering contributions and pensions (i.e. a non-self-financing portfolio) makes our model more general than the usual portfolio optimisation framework with jumps.

We follow Battocchio et al. (2007) where the optimal portfolio for a pension fund is computed in a simple framework with: (i) constant contributions and pensions, (ii) a riskless asset with constant return, and (iii) a risky asset whose price follows a geometric Brownian motion. Here, we take into account the point of view of a consumer/investor who optimises the expected utility of his/her consumption during his/her lifetime and we model the risky asset return with a Lévy process. This setting allows us to take into account a more realistic behaviour of the asset returns by modelling asymmetry and leptokurtosis (see, e.g., the contributions of Merton, 1990; Aït-Sahalia and Jacod, 2010; Carr et al., 2002; and Øksendal and Sulem, 2010). As shown in the last section, taking jumps into account has a dramatic effect on asset allocations.

In our framework, we merge two relevant aspects of asset allocation: (i) the non-self-financing wealth and (ii) the risk of negative jumps on asset returns. First, we emphasise that an agent must base his/her strategy on the whole wealth that also takes into account the future pension income. This strategy implies higher holdings of risky assets and more consumption with respect to the basic Merton model, mainly during the so-called accumulation phase. Second, the presence of negative jumps reduces the amount of wealth invested in risky assets since they are less attractive, confirming a result already shown in Øksendal and Sulem (2010).

When Lévy processes are used in finance, stock price dynamics are generally modelled as standard exponentials. This is especially useful for calibration. However, portfolio theories are mainly based on stochastic exponentials. This paper combines these two approaches to solve the optimal portfolio problem and apply the theoretical solution to real data. We obtain the optimal portfolio in quasi-closed form as the solution of an ordinary algebraic equation. Closed form solutions are available for particular choices of the underlying Lévy process. By studying two examples of Lévy processes, switching back and forth between the standard and stochastic exponentials, and estimating our model on historical S&P500 Index data, we show that the optimal investment in the risky asset for a pension scheme is up to 50% lower than the optimal investment under the Gaussian approach.

We show that our framework which takes into account a pension scheme can be traced back to the basic Merton's through a suitable mathematical simplification which is shown to coincide with the so-called retrospective mathematical reserve, i.e. the compounded sum of all the contributions and pensions paid/received by the investor. The wealth net of this reserve (called modified wealth in the paper) behaves exactly like the wealth in the Merton's basic model. Thus, we show that the optimal portfolio share is actually constant with respect to the modified wealth (like in Merton), but it is not constant with respect to the investor's (original) wealth and we show the difference which makes our model more general. In particular, we show that during the so-called accumulation phase (when contributions are paid) the investment in the risky asset must be higher that obtained in a model without any pension scheme.

To sum up, this paper proposes a framework for optimising the composition of wealth and the consumption of an agent endowed with a pension in the context of a market with jumps. In this framework, the HJB equation is solved in the presence of both jumps and mortality. We obtain the optimal weight in the risky asset and we also obtain and study in detail the optimal consumption. Specifically, we relate the behaviour of the optimal consumption to that of the force of mortality. Then, we examine the small jumps approximation. Next, the optimal consumption is computed under the Gompertz–Makeham assumption. We also study the positivity of the aggregate wealth. This paper also provides a stochastic (Doleans–Dade) exponential version of the Variance Gamma model. It extends the model of Aït-Sahalia et al. (2009) by shifting the contribution of negative jumps. This paper also relates the stochastic exponential and classic exponential approaches, so that it is possible to show how to link the solution to the HJB equation to a real-world calibration. Finally, an illustration on the optimal weight and consumption is conducted. In the latter case, this analysis is conducted with respect to mortality parameters. Note that after calibrating the parameters of the VG and EACH models, a Vuong test was performed to be able to discriminate between the two models.

The paper is organised as follows. Section 1 presents the setting that consists of both the financial and the actuarial frameworks and the optimisation problem for a consumer/investor during his/her lifetime. In Section 2, the optimal portfolio is computed in quasi-explicit form, and the optimal consumption is computed in closed form. Section 3 explains how it is possible to switch back and forth between the standard and stochastic exponential representations, which is a condition for fully exploiting the framework with real-world data. This section also presents two models for asset returns based on Lévy processes that allow us to match the properties of the empirical asset returns. We show in Section 4 how the estimation of model parameters can be conducted, and we give numerical computations of the optimal portfolio.

2. The setting

We start this section by describing the setting of this paper. After presenting the financial market setting, we give both the actuarial framework for the investor's mortality and the model for the dynamics of his/her wealth. Then, we formulate the optimisation problem solved in the following sections.

2.1. Asset dynamics

We consider an infinite-horizon and continuous-time financial market where two assets are listed: (i) a risk-free asset, and (ii) a non-dividend-paying risky asset, typically a stock index. The initial time is t_0.

The price S_0 of the risk-free asset verifies the ordinary differential equation

$$\frac{dS_0(t)}{S_0(t)} = rdt, \tag{1}$$

where r is the constant instantaneous risk-free interest rate, and the initial price $S_0(t_0)$ is known.

The risky asset's price S is modelled by a stochastic differential equation (SDE) including a general Lévy process:

$$\frac{dS(t)}{S(t)} = \mu_L dt + \sigma dB(t) + dL(t). \tag{2}$$

The parameters μ_L and σ are constant, B is a Wiener process with $B(t_0) = 0$, and L is a general Lévy process with $L(t_0) = 0$. Its jumps belong to $]-1, +\infty[$, and its Lévy measure, or the arrival rate