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Examining macroeconomic models through the lens of asset pricing

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ABSTRACT

We develop new methods for representing the asset-pricing implications of stochastic general equilibrium models. We provide asset-pricing counterparts to impulse response functions and the resulting *dynamic value decompositions (DVDs)*. These methods quantify the exposures of macroeconomic cash flows to shocks over alternative investment horizons and the corresponding prices or investors' compensations. We extend the continuous-time methods developed in Hansen and Scheinkman (2012) and Borovička et al. (2011) by constructing discrete-time, state-dependent, shock-exposure and shock-price elasticities as functions of the investment horizon. Our methods are applicable to economic models that are nonlinear, including models with stochastic volatility.

1. Introduction

It is standard practice to represent implications of dynamic macroeconomic models by showing how featured time series respond to shocks. Alternative current period shocks influence the future trajectory of macroeconomic processes such as consumption, investment or output, and these impacts are measured by impulse response functions. From an asset pricing perspective, these functions reflect the *exposures* of the underlying macroeconomic processes to shocks. These exposures depend on how much time has elapsed between the time the shock is realized and time of its impact on the macroeconomic time series under investigation. Changing this gap of time gives a trajectory of exposure elasticities that we measure. In this manner we build *shock-exposure* elasticities that are very similar to and in some cases coincide with impulse response functions.

In a fully specified dynamic stochastic general equilibrium model, exposures to macroeconomic shocks are priced because investors must be compensated for bearing this risk. To capture this compensation, we produce pricing counterparts to impulse response functions by representing and computing *shock-price* elasticities implied by the structural model. These prices are the risk compensations associated with the shock exposures. The shock-exposure and shock-price elasticities provide us with dynamic value decompositions (DVDs) to be used in analyzing alternative structural models that have valuation implications. Quantity dynamics reflect the impact of *current* shocks on future distributions of a macroeconomic process, while pricing dynamics reflect the current period compensation for the exposure to *future* shocks.

In our framework the shock-exposure and shock-price elasticities have a common underlying mathematical structure. We build processes that grow or decay stochastically in a geometric fashion. They capture the compounding of the discount and/or growth rates over time. We construct the shock elasticities that measure the intertemporal responses to changing exposures of these processes to alternative shocks. We interpret the objects of interest as 'elasticities' because they reflect the sensitivity of the logarithm of expected returns or expected cash flows to a change in the exposure to a shock normalized to have a unit standard deviation. The shock elasticities are state-dependent and reflect the nonlinearities of the dynamic model. We provide an abstract construction of the elasticities and ways to compute them in practice, including tractable frameworks suitable for applications in dynamic, stochastic general equilibrium (DSGE) modeling.

While these elasticities have not been explored in the quantitative literature in macroeconomics, they have antecedents in the asset pricing literature. The intertemporal structure of risk premia has been featured in the term structure of interest rates, but this literature purposefully abstracts from the pricing of stochastic growth components in the macroeconomy. Recently Lettau and

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Wachter (2007) and Hansen et al. (2008) have explored the term structure of risk premia explicitly in the context of equity claims that grow over time. Risk premia reflect contributions from exposures and prices of those exposures. Here we build on an analytical framework developed in Alvarez and Jermann (2005), Hansen and Scheinkman (2009, 2012) and Borovička et al. (2011) to distinguish exposure elasticities and price elasticities.

The shock elasticities are also conceptually close to nonlinear versions of impulse response functions, introduced in Gallant et al. (1993); Koop et al. (1996) or Gourieroux and Jasiak (2005). In a loglinear framework, the shock elasticities exactly correspond to impulse response functions familiar from VAR analysis applied to the logarithms of stochastic growth or discount factor processes. In nonlinear models, our elasticities trace out changes in conditional expectations of future quantities in response to a *marginal* change in shock exposures. We design our approach to give a direct link to familiar characterizations of risk prices extended to multiple payoff horizons. We provide a way to operationalize the continuous-time formulations in Hansen and Scheinkman (2012), Borovička et al. (2011) and Hansen (2012) in a discrete-time setting.

In Section 2, we develop the concept of shock elasticities in a general framework. The shock elasticities arise naturally in decompositions of risk premia into the contribution of shocks at different horizon. In Section 3, we show that similar decompositions can be employed in deconstructing entropy measures of Backus et al. (2011) used to analyze the dynamics of the stochastic discount factor. An important goal of this paper is a tractable implementation of DVDs. We therefore devote Sections 4 and 5 to the discussion of methods that solve for approximate dynamics in a broad class of DSGE models. We pay particular attention to the approximation of recursive preferences of Kreps and Porteus (1978) and Epstein and Zin (1989) since these preferences play a prominent role in the asset pricing literature. We show that a second-order perturbation approximation of the DSGE models derived using the series expansion methods can be nested within an exponential–quadratic framework in which the shock elasticities are available in quasi-analytical form. We introduce this framework in Section 6 and discuss details of the solution in the Appendix. We also provide Matlab codes for the computation of the shock elasticities in models solved by Dynare.

Finally, in Section 7, we illustrate the developed tools in measuring shock exposures and model-implied prices of exposure to those shocks in a model with physical and intangible capital constructed by Ai et al. (2012). A reader immediately interested in the applicability of the introduced methods can read this section directly after, or in parallel to, Section 2.

2. Analytical framework

In this section we describe some basic tools for valuation accounting, by which we provide measures of shock exposures and shock prices for alternative investment horizons. In our framework the shock-exposure and shock-price elasticities have a common underlying mathematical structure. Let M be a process that grows or decays stochastically in a geometric fashion. It captures the compounding discount and/or growth rates over time in a stochastic fashion and is constructed from an underlying Markov process X. Let W be a sequence of independent and identically distributed standard normal random vectors. The common ingredient in our analysis is the ratio:

$$\varepsilon_m(x,t) = \alpha_h(x) \cdot \frac{E\left[M_t W_1 \mid X_0 = x\right]}{E\left[M_t \mid X_0 = x\right]} \tag{1}$$

where x is the current Markov state and α_h selects the linear combination of the shock vector W_1 of interest. The state dependence in α_h allows for analysis of stochastic volatility. We interpret this entity as a "shock elasticity" used to quantify the date t impact on values of exposure to the shock $\alpha_h(x)W_1$ at date one.

We add more structure to this formulation, by considering dynamic systems of the form

$$X_{t+1} = \psi(X_t, W_{t+1}) \tag{2}$$

where W is a sequence of independent shocks distributed as a multivariate standard normal. In much of what follows we will focus on stationary solutions for this system. By imposing appropriate balanced growth restrictions, we suppose that the logarithms of many macroeconomic processes that interest us grow or decay over time and can be represented as:

$$Y_t = Y_0 + \sum_{s=0}^{t-1} \kappa(X_s, W_{s+1}) \tag{3}$$

where Y_0 is an initial condition, which we will set conveniently to zero in much of our discussion. A typical example of the increment to this process is

$$\kappa(X_s, W_{s+1}) = \beta(X_s) + \alpha(X_s) \cdot W_{s+1}$$

where the function β allows for nonlinearity in the conditional mean and the function α introduces stochastic volatility. We call such a process Y an *additive functional* since it accumulates additively over time, and can be built from the underlying Markov process X provided that W_{t+1} can be inferred from X_{t+1} and X_t. By a suitable construction of the state vector, this restriction can always be met. The state vector X thus determines the dynamics of the increments in Y. When X is stationary Y has stationary increments.

While the additive specification of Y is convenient for modeling logarithms of economic processes, to represent values of uncertain cash flows it is necessary to study levels instead of logarithms. We therefore use the exponential of an additive functional, $M = \exp(Y)$, to capture growth or decay in levels. We will refer to M as a *multiplicative functional* represented by κ or sometimes the more restrictive specification (α, β).

In what follows we will consider two types of multiplicative functionals, one that captures macroeconomic growth, denoted by G, and another that captures stochastic discounting, denoted by S. The stochastic nature of discounting is needed to adjust consumption processes or cash flows for risk. Thus S, and sometimes G as well, are computed from the underlying economic model to reflect equilibrium price dynamics. For instance, G might be a consumption process or some other endogenously determined cash flow, or it might be an exogenously specified technology shock process that grows through time. The interplay between S and G will dictate valuation over multi-period investment horizons.

Our aim is to use a structural stochastic equilibrium model with identified macroeconomic shocks to *deconstruct* the asset-pricing implications. Such a model will imply a stochastic discount factor process S and benchmark stochastic growth processes. While for empirical purposes the pricing implications are conveniently captured by the stochastic discount factor process, with DVD methods we use the identified macroeconomics shocks as vehicles for interpreting the resulting pricing implications. These methods measure two things: (i) how exposed are future macroeconomic processes to next-period shocks, and (ii) what are the implied prices for these shock exposures. Measurements (i) are very closely related to familiar impulse response functions. Our use of measurements (ii) reflects a more substantive departure from common practice in the macroeconomics literature. We view these latter measurements as the pricing counterparts to impulse response functions.

2.1. One-period asset pricing

It is common practice in the asset pricing literature to represent prices of risk in terms of expected return on an investment per