Judgment scales and consistency measure in AHP

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Abstract

The Analytic Hierarchy Process (AHP) is widely used method in multiple-attribute decision making. In the recent literature many authors used different judgment scales which influenced the results and decisions. In this paper the author reviews and discusses effects of utilization of various judgment scales on priority estimation in AHP. There has been studies that have been concerned with the comparison of judgment scales but there were no studies concerned with consistency measures that are needed. The goal of this paper is to compare and discuss the application of various judgment scales on the results in particular practical example that has been used in previous paper by Saaty (2003). Thus the focus of the paper is to analyze the impact of using different judgment scales on the resulting priorities and consistency to default scale as proposed by Saaty. Results suggest that judgment scales have a profound impact on criteria priorities but not on ranking of criteria. However, the consistency varies among applied judgment scales. Authors calculated the values of random index needed for calculation of the consistency index in AHP for all concerned scales. Based on them the consistency index was computed and compared. Both consistent and inconsistent Saaty matrices were used for comparison.

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1. Introduction

The analytic hierarchy process (AHP) is a decomposition multiple-attribute decision making (MADM) method. It was developed by Saaty (1977), who proposed a method that can represent human decision making process and help to achieve better judgments based on hierarchy, pair-wise comparisons, judgment scales, allocation of criteria weights and selection of the best alternative from a finite number of variants by calculation of their utility functions. Subsequently, there has been a growth of applications and mathematical development to this methodology. These developments were focused on different parts of the method. A significant development has been made by Saaty (1996) who presented a more general approach to AHP which he called the analytic network process (ANP). Other scholars have suggested new judgment scales or they expand the method by fuzzy logic and group decision making (Ishizaka & Labib, 2011).

The goal of this paper is to analyze judgment scales developed for AHP and their influence on results of AHP decision making example. The aim of this approach is to analyze inconsistent and consistent pair-wise comparison matrices to get a better understanding of how changes in scales can influence results and consistency at the same time. Suggestions of how these scales and results of this study can be applied can be found in the discussion and conclusion.

2. Literature review

Since its introduction, AHP has been widely used (Taslicali & Ercan, 2006). AHP is a multi-criteria decision making (MCDM) method that helps decision-maker to face a complex problem with multiple conflicting and subjective criteria (e.g. location or investment selection, projects ranking, etc).

2.1. Summary of the AHP

Basically the method uses following structure: problem modeling, weights valuation, weights aggregation and sensitivity analysis. AHP has the advantage of permitting a hierarchical structure of the criteria, which provides users with a better focus on specific criteria and sub-criteria when allocating the weights. This step is important, because a different structure may lead to a different final ranking. When setting up the AHP hierarchy with a large number of elements, the decision maker should attempt to arrange these elements in clusters so they do not differ in extreme ways.

Psychologists argue that it is easier and more accurate to express one's opinion on only two alternatives than simultaneously on all the alternatives (Ishizaka & Labib, 2011). This also allows consistency check of different pair-wise comparisons. AHP uses a ratio scale, which, contrary to methods using interval scales, requires no units in the comparison. The judgment is a relative value or a quotient a/b of two quantities a and b having the same units (intensity, meters, utility, etc). The decision maker does not need to provide a numerical judgment; instead a relative verbal appreciation is sufficient. The results of paired comparisons for n attributes is organized into positive reciprocal $n \times n$ matrix Psychologists argue that it is easier and more accurate to express one's opinion on only two alternatives than simultaneously on all the alternatives (Saaty, 1977). This also allows consistency check of different pair-wise comparisons. AHP uses a ratio scale, which, contrary to methods using interval scales, requires no units in the comparison. The judgment is a relative value or a quotient a / b of two quantities a and b having the same units (intensity, meters, utility, etc). The decision maker does not need to provide a numerical judgment; instead a relative verbal appreciation is sufficient. The results of paired comparisons for n attributes are organized into positive reciprocal $n \times n$ matrix $\mathbf{S} = \left(s_{ij} \right)$ as follows

$$\mathbf{S} = \begin{pmatrix} 1 & s_{12} & ... & s_{1n} \\ 1/s_{12} & 1 & ... & s_{2n} \\ ... & ... & ... & ... \\ 1/s_{1n} & 1/s_{2n} & ... & 1 \end{pmatrix}.$$

(1)