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Comparative performance of meta-heuristic algorithms for mass minimisation of trusses with dynamic constraints

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ABSTRACT

This paper investigates the search performances of various meta-heuristics (MHs) for solving truss mass minimisation with dynamic constraints. Several established MHs were used to solve five truss optimisation problems. The results obtained from using the various MHs were statistically compared based upon convergence rate and consistency. It was found that the best optimisers for this design task are evolution strategy with covariance matrix adaptation (CMAES) and differential evolution (DE). Furthermore, the best penalty function technique was discovered while four penalty function techniques assigned with several parameter settings were used in combination with the five best optimisers to solve the truss optimisation problems.

Introduction

A truss is one of the most used structures in engineering applications due to its simplicity and low cost for construction. Under working conditions, such a structure is subject to multiple types of loads, which can be categorised as being static or dynamic. The structure under dynamic excitation may cause several undesirable vibration phenomena including structural resonance. As a result, truss designers have to find ways to prevent or suppress structural vibration. The problem of truss minimisation subject to natural frequency constraints has been researched [1–6], and it has been found that predefined frequency constraints can prevent vibration resonance from certain frequency bands of dynamic loads. Since its introduction, this special design problem has been problematic because its feasible region is non-convex while the boundaries are highly non-linear [1–3]. This has led to difficulty in the use of gradient-based optimisers. Alternatively, meta-heuristics (MHs) are known to be a better choice. Many researchers in the field of structural optimisation have been interested in the investigation of solving this kind of optimisation problem using MHs. Normally, the problem has an objective in minimising structural mass with multiple frequency constraints while using design variables such as topology [4], sizing [1,7], and a combination of shape and size [2,3,5,6].

Meta-heuristic algorithms are widely used for various kinds of optimisation design problems, particularly for engineering applications, due to their derivative-free and global optimisation capabilities. These optimisers are robust and can be used to solve almost any kind of optimisation due to the nature of soft computing. However, a lack of search consistency is unavoidable during the MH search because of procedural randomisation. Also, there is likely to be a slow convergence rate if parameter settings, which are problem-dependent, are not properly assigned. Over the last few decades, numerous MHs have been developed, improved upon, and successfully implemented on a wide variety of optimisation problems [8–14]. New MHs can be proposed as hybridisations of existing algorithms [9–10,13,14] or have been introduced as new search concepts [15–18]. Even with thousands of algorithms in the literature, it can be said that there is no single MH that can outperform other MHs for all design problems. Consequently, it is always useful to compare the performance of a number of well-established and newly developed MHs for solving a newly introduced design problem or a problem that remains difficult to solve. For truss mass minimisation with frequency constraints, there are a number of MHs that have been successfully used including a charged system search algorithm [4], a hybrid version of the charged system search algorithm [3], a genetic algorithm [6,7], a particle swarm algorithm [1,19], a harmony search [2,5] and a firefly algorithm [5]. Nevertheless, since those researchers conducted their work independently, they presented a new meta-heuristic and compared it only with the previously used optimisers in the literature. Usually, the new method was run many times and the best results were taken to show and compare. This is not an appropriate way to compare search performances of MHs since it

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is unlikely that those optimisers can get the same results with several runs due to randomisation in the process. There are a large number of MHs that have been overlooked and not implemented, while there have been several penalty function techniques being used such as the traditional exterior penalty function technique [2], the fuzzy set theory technique [20], the Kaveh–Zolghadr technique [3], and the Murata–Kim–Sugei technique [21]. The comparison of their effectiveness would be useful for future engineers and researchers.

In this comparative study, various MHs (mostly well-established), have been employed to solve a number of traditional test problems for truss optimisation with frequency constraints. Twenty-four MHs were used to solve five test problems. The top five best performers were then used in combination with four penalty function techniques and various sets of their internal parameters to solve the test problems in order to examine the effectiveness of those penalty function techniques. Each optimiser was used to solve each test problem a number of times, while the results obtained from using the various methods were statistically compared. The best meta-heuristics and the most effective penalty function methods will be discussed. What follows in this paper is organised such that: Section 'Test problems of truss shape and sizing optimisation' gives details of the test problems for truss mass minimisation with frequency constraints. Section 'Meta-heuristics' shows the comparative performance of the 24 meta-heuristic optimisers on solving the five test problems. The top five best optimisers are then used to examine the effectiveness of the penalty function techniques while the comparative results of this study are given in section 'Comparative study of various meta-heuristic algorithms'. Finally, concluding remarks are provided in section 'Comparative study of various penalty function techniques'.

Test problems of truss shape and sizing optimisation

A typical truss optimisation problem with dynamic constraints can be posed as

$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & \omega_i \le \omega_{i,all} \quad \text{for } i \in I \\ & \omega_j \ge \omega_{j,all} \quad \text{for } j \in J \\ & \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{aligned} \tag{1}$$

where $\mathbf{x}$ is a vector containing design variables having lower and upper bounds as $\mathbf{x}_L$ and $\mathbf{x}_U$ respectively, f is an objective function, ω_i is the i-th mode natural frequency of a truss, $\omega_{i,all}$ is an allowable frequency for the i-th mode natural frequency, I and J are the sets of mode numbers to be specified by a designer.

The objective function is structural mass whereas design variables include, in this work, sizing and shape variables. The sizing design variables include truss element cross-sectional areas while the shape design variables are nodal positions of the structure. It is commonplace that most if not all of MHs can handle optimisation problems with bound constraints. For the design problem (1), a penalty function, which will be detailed later, is adopted to handle the dynamic constraints.

Five test problems of shape and sizing optimisation of trusses with frequency constraints commonly found in the literature will be employed in this study. The design problems were assigned to minimise structural mass subject to multiple frequency constraints, and are detailed as follows:

Case I: Sizing design of a 2D 10 bar truss

The truss structure is illustrated in Fig. 1 [1,3,5,6]. The structure is subject to non-structural mass of 454 kg at each free node. The design variables include all simple bar element cross section areas. Material density and modulus of elasticity are 2770.0 kg/m^3 and 6.98×10^{10} N/m^2 respectively. The sizing optimisation is presented to minimise structural mass subject to natural frequency constraints which can be expressed as follows:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) = \text{structural mass} \\ \text{s.t.} \quad & \omega_1 \ge 7 \text{ Hz} \\ & \omega_2 \ge 15 \text{ Hz} \\ & \omega_3 \ge 20 \text{ Hz} \\ & 0.645 \le A_i \le 50 \text{ cm}^2 \end{aligned}$$

where $\mathbf{x}^T = \{x_1,\ldots,x_{10}\} = \{A_1,\ldots,A_{10}\}$ is a design vector, $f(\mathbf{x})$ is structural mass. The variables ω_1, ω_2 and ω_3 are the natural frequencies for the first, second and third modes respectively. A_i are cross-sectional areas of all bar elements.

Case II: Shape and sizing design of a 2D 37 bar truss

The truss structure is illustrated in Fig. 2 [1,2,5,6,19]. The structure is subject to non-structural mass of 10 kg at each free node of the lower chord. The elements of the lower chord were set as bar elements with unchanged cross-sectional area of 40 cm^2 while the other were set as bar elements with initial cross-sectional area 1 cm^2. The design variables include all bar element cross-sectional areas (except the lower chord bar elements) and y-direction of nodal positions of the upper chord. The design variables are treated so to have a symmetrical structure with respect to the y axis. Material density and modulus of elasticity are 7800.0 kg/m^3 and 2.1×10^{11} N/m ^{2}, respectively. The shape and sizing optimisation problem is presented to minimise structural mass subject to frequency constraints which can be expressed as follows:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) = \text{structural mass} \\ \text{s.t.} \quad & \omega_1 \ge 20 \text{ Hz} \\ & \omega_2 \ge 60 \text{ Hz} \\ & \omega_3 \ge 60 \text{ Hz} \\ & 1 \le A_i \le 10 \text{ cm}^2 \\ & 0.1 \le Y_i \le 3 \text{ m} \end{aligned}$$

where $\mathbf{x}^T = \{x_1,\ldots,x_{19}\}$ is a design vector. Truss cross-sectional areas are assigned as the first 14 elements of the vector while the nodal positions are assigned as the 15th- 19th elements. The variables ω_1, ω_2 and ω_3 are the natural frequencies for the first, second and third modes respectively. A_i are element cross-sectional areas. Y_i are the y-direction of positions of the upper chord nodes (changes in Y_i are symmetric about y axis).

Case III: Sizing design of a 3D 72 bar space truss

The truss structure is illustrated in Fig. 3 [1,3,5,7]. Four non-structural masses of 2270 kgs are attached to the top nodes. The design variables include all bar element cross-sectional areas which were divided into 16 groups according to Table 1. Material density and modulus of elasticity are 2770.0 kg/m^3 and 6.98×10^{10} N/m^2, respectively. The sizing design problem is assigned to minimise structural mass subject to frequency constraints, which can be expressed as follows:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) = \text{structural mass} \\ \text{s.t.} \quad & \omega_1 = 4 \text{ Hz} \\ & \omega_2 \geqslant 6 \text{ Hz} \\ & 0.645 \le A_i \leqslant 30 \text{ cm}^2 \end{aligned}$$

where $\mathbf{x}^T = \{x_1,\ldots,x_{16}\}$ is a design vector. ω_1 and ω_2 are the first and second mode natural frequencies respectively. A_i are the cross-sectional areas.

Case IV: Shape and sizing design of a 3D 52 bar dome truss

The truss structure is illustrated in Fig. 4 [1–3,5,6,19]. The structure has non-structural masses of 50 kg at each free node. The