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# A concept of fuzzy input mix-efficiency in fuzzy DEA and its application in banking sector

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ABSTRACT

Data envelopment analysis (DEA) is a linear programming based non-parametric technique for evaluating the relative efficiency of homogeneous decision making units (DMUs) on the basis of multiple inputs and multiple outputs. There exist radial and non-radial models in DEA. Radial models only deal with proportional changes of inputs/outputs and neglect the input/output slacks. On the other hand, non-radial models directly deal with the input/output slacks. The slack-based measure (SBM) model is a non-radial model in which the SBM efficiency can be decomposed into radial, scale and mix-efficiency. The mix-efficiency is a measure to estimate how well the set of inputs are used (or outputs are produced) together. The conventional mix-efficiency measure requires crisp data which may not always be available in real world applications. In real world problems, data may be imprecise or fuzzy. In this paper, we propose (i) a concept of fuzzy input mix-efficiency and evaluate the fuzzy input mix-efficiency using $\alpha$ – cut approach, (ii) a fuzzy correlation coefficient method using expected value approach which calculates the expected intervals and expected values of fuzzy correlation coefficients between fuzzy inputs and fuzzy outputs, and (iii) a new method for ranking the DMUs on the basis of fuzzy input mix-efficiency. The proposed approaches are then applied to the State Bank of Patiala in the Punjab state of India with districts as the DMUs.



## 1. Introduction

Data envelopment analysis (DEA), proposed by Charnes, Cooper, and Rhodes (1978), is a linear programming based non-parametric method for evaluating the relative efficiency of homogeneous decision making units (DMUs) on the basis of multiple inputs and multiple outputs. The popularity of DEA is due to its ability to measure relative efficiencies of DMUs without prior weights on the inputs and outputs. There are two types of models in DEA: radial and non-radial. Radial model is represented by the CCR model (Charnes et al., 1978), the first DEA model. Basically, it deals with proportional changes of inputs or outputs. The CCR efficiency score reflects the proportional maximum input (output) reduction (expansion) rate which is common to all inputs (outputs). However, in real world businesses, not all inputs (outputs) behave in the proportional way. Also a radial model neglects slacks in inputs/outputs while reporting the efficiency score. In many cases, we find a lot of remaining non-radial slacks. So, if these slacks have an important role in evaluating the efficiency, the radial approaches may mislead the decision when we utilize the efficiency score as the only index for evaluating performance of DMUs. In contrast, the non-radial model is represented by the slack-based measure (SBM) (Tone, 2001), which put aside the assumption of proportionate changes in inputs and outputs, and deal with slacks directly. Also the SBM model assesses the efficiency of the input or output mix as well as it assesses the overall level of efficiency. It has three variations (i) input-oriented, (ii) output-oriented, and (iii) non-oriented. For details of comparison between radial and non-radial measure, see Avkiran, Tone, and Tsutsui (2008) along with the shortcomings for both the CCR and the SBM models. Tone (1998) suggests that the results from both the CCR and the SBM models can be used to evaluate the mix-efficiency. The mix-efficiency is a measure to estimate how well the set of inputs are used (or outputs are produced) together (Herrero, Pascoe, & Mardle, 2006; Asbullah, 2010). Tone (1998) presented input mix-efficiency and output mix-efficiency by using input-oriented and output-oriented variations of both CCR and SBM models.

Conventional mix-efficiency measure requires crisp input and output data, which may not always be available in real world applications. Actually, in real world problems, inputs and outputs are often imprecise or fuzzy. So, in order to calculate mix-efficiency with imprecise or fuzzy data, we propose the concept of fuzzy input mix-efficiency (FIME). For measuring FIME, we propose the input-oriented fuzzy CCR model ($FCCR_I$) and input-oriented fuzzy

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**Table 1**
Input and output data with $\theta_I^k$, $\rho_I^k$ and $\psi_I^k$.

| DMUS | Input 1 | Input 2 | Output 1 | Output 2 | $\theta_I^k$ | Rank | $\rho_I^k$ | Rank | $\psi_I^k$ | Rank |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 151 | 100 | 90 | 1.000000 | 1 | 1.000000 | 1 | 1.000000 | 1 |
| 2 | 19 | 131 | 150 | 50 | 1.000000 | 1 | 1.000000 | 1 | 1.000000 | 1 |
| 3 | 25 | 160 | 160 | 55 | 0.882708 | 8 | 0.852165 | 8 | 0.965399 | 8 |
| 4 | 27 | 168 | 180 | 72 | 1.000000 | 1 | 1.000000 | 1 | 1.000000 | 1 |
| 5 | 22 | 158 | 94 | 66 | 0.763499 | 12 | 0.755612 | 11 | 0.989670 | 5 |
| 6 | 55 | 255 | 230 | 90 | 0.834771 | 10 | 0.703764 | 12 | 0.843062 | 12 |
| 7 | 33 | 235 | 220 | 88 | 0.901961 | 7 | 0.894835 | 6 | 0.992100 | 4 |
| 8 | 31 | 206 | 152 | 80 | 0.796334 | 11 | 0.773958 | 10 | 0.971901 | 7 |
| 9 | 30 | 244 | 190 | 100 | 0.960392 | 4 | 0.904642 | 5 | 0.941950 | 9 |
| 10 | 50 | 268 | 250 | 100 | 0.870647 | 9 | 0.780509 | 9 | 0.896471 | 11 |
| 11 | 53 | 306 | 260 | 147 | 0.955098 | 6 | 0.866137 | 7 | 0.906856 | 10 |
| 12 | 38 | 284 | 250 | 120 | 0.958204 | 5 | 0.936020 | 4 | 0.976848 | 6 |

Source of input and output data: Cooper, Seiford and Tone, 2007.

**Table 2**
The fuzzified data in terms of TFNs.

| DMUS | Input 1 (I1) | Input 2 (I2) | Output 1 (O1) | Output 2 (O2) |
|---|---|---|---|---|
| 1 | (16,20,22) | (150,151,152) | (95,100,102) | (87,90,94) |
| 2 | (18,19,20) | (130,131,132) | (149,150,151) | (46,50,52) |
| 3 | (23,25,28) | (158,160,162) | (158,160,163) | (53,55,56) |
| 4 | (26,27,29) | (165,168,169) | (177,180,181) | (70,72,75) |
| 5 | (20,22,25) | (155,158,162) | (90,94,98) | (63,66,68) |
| 6 | (52,55,59) | (250,255,259) | (222,230,235) | (83,90,95) |
| 7 | (30,33,34) | (234,235,236) | (210,220,225) | (81,88,90) |
| 8 | (27,31,33) | (202,206,208) | (151,152,155) | (75,80,84) |
| 9 | (26,30,35) | (240,244,247) | (188,190,193) | (99,100,101) |
| 10 | (47,50,54) | (262,268,271) | (246,250,252) | (94,100,108) |
| 11 | (50,53,56) | (300,306,309) | (255,260,264) | (143,147,152) |
| 12 | (30,38,42) | (283,284,285) | (246,250,254) | (116,120,123) |

SBM model ($FSBM_I$) with fuzzy input and fuzzy output data. Several approaches have been developed to deal with imprecise or fuzzy data in DEA. Sengupta (1992) applied principle of fuzzy set theory to introduce fuzziness in the objective function and the right-hand side vector of the conventional DEA model. Guo and Tanaka (2001) used the ranking method and introduced a bi-level programming model. Lertworasirikul (2001) developed a method in which the inputs and outputs were firstly defuzzified and then the model was solved using $\alpha$-cut approach. There are some other approaches based on $\alpha$-cut which can be found in Meada, Entani, and Tanaka (1998), Kao and Liu (2000a) and Saati Mohtadi, Memariani, and Jahanshahloo (2002). Lertworasirikul, Fang, Jeffrey, Joines, and Nuttle (2003) proposed a possibility DEA model for fuzzy DEA (FDEA). Kao and Liu (2000a, 2000b, 2003, 2005) transformed fuzzy input and fuzzy output into intervals by using $\alpha$-level sets and built a family of crisp DEA models for the intervals. Liu (2008) and Liu and Chuang (2009) developed a fuzzy DEA/AR model for the selection of flexible manufacturing systems and the assessment of university libraries respectively. Zhou, Lui, Ma, Liu, and Liu (2012) proposed a generalized fuzzy data envelopment model with assurance regions, whose lower and upper bounds at given levels could be obtained. Entani, Maeda, and Tanaka (2002) and Wang, Greatbanks, and Yang (2005) also changed fuzzy input and fuzzy output data into intervals by using$\alpha$-level sets, but suggested two different interval DEA models. Dia (2004) proposed a FDEA model based on fuzzy arithmetic operations and fuzzy comparisons between fuzzy numbers. The model requires the decision maker to specify a fuzzy aspiration level and a safety $\alpha$-level so that the FDEA model could be transformed into a crisp DEA model for solution. Wang, Luo, and Liang (2009) constructed two FDEA models from the perspective of fuzzy arithmetic to deal with fuzziness in input and output data in DEA. The two FDEA models were both formulated as linear programs and could be solved to determine fuzzy efficiencies of DMUs. Jahanshahloo, Soleimani-damaneh, and Nasrabadi (2004) extended a slack-based measure (SBM) of efficiency in DEA to fuzzy settings and developed a bi-objective nonlinear DEA model for FDEA. Among all the approaches to solve FDEA, the most popular approach is $\alpha$-cut approach. Hatami-Marbini, Saati, and Makui (2010) introduced two virtual DMUs called ideal DMU (IDMU) and anti-ideal DMU (ADMU) with fuzzy inputs-outputs, and evaluated efficiency of DMUs by FDEA. Hatami-Marbini, Saati, and Tavana (2010) presented a four-phase FDEA framework based on the theory of displaced ideal. Wang and Chin (2011) proposed a "fuzzy expected value approach" for DEA in which fuzzy inputs and fuzzy outputs are first weighted respectively, and their expected values then used to measure the optimistic and pessimistic efficiencies of DMUs in fuzzy environments. Hsiao, Chern, Chiu, and Chiu (2011) proposed the fuzzy super-efficiency slack-based measure DEA model using $\alpha$-cut approach and analyze the operational performance of 24 commercial banks facing problems on loan and investment parameters with vague characteristics. Majid Zerafat Angiz, Emrouznejad, and Mustafa (2012) introduced an alternative linear programming model that can include some uncertainty information from the intervals within the $\alpha$-cut approach and proposed the concept of "local $\alpha$-level" to develop a multi-objective linear programming to measure the efficiency of DMUs under uncertainty.

In this paper, we use $\alpha$-cut approach to solve $FCCR_I$ and $FSBM_I$. Then, the results of these models are applied to calculate FIME. We propose a new method for calculating the fuzzy correlation

**Table 3**
The expect intervals and the corresponding expected values of the fuzzy correlation coefficients.

| | $r_L$ | | | | $r_R$ | | | | $r^{EV}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | I1 | I2 | O1 | O2 | I1 | I2 | O1 | O2 | I1 | I2 | O1 | O2 |
| I1 | 1.0000 | 0.8404 | 0.8314 | 0.6505 | 1.0000 | 0.8809 | 0.8472 | 0.7175 | 1.0000 | 0.8606 | 0.8393 | 0.6840 |
| I2 | 0.8404 | 1.0000 | 0.8862 | 0.8737 | 0.8809 | 1.0000 | 0.8882 | 0.8839 | 0.8606 | 1.0000 | 0.8872 | 0.8788 |
| O1 | 0.8314 | 0.8862 | 1.0000 | 0.6753 | 0.8472 | 0.8882 | 1.0000 | 0.6955 | 0.8393 | 0.8872 | 1.0000 | 0.6854 |
| O2 | 0.6505 | 0.8737 | 0.6753 | 1.0000 | 0.7175 | 0.8839 | 0.6955 | 1.0000 | 0.6840 | 0.8788 | 0.6854 | 1.0000 |