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Super-efficiency based on a modified directional distance function

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ABSTRACT

The problem of infeasibility arises in conventional radial super-efficiency data envelopment analysis (DEA) models under variable returns to scale (VRS). To tackle this issue, a Nerlove–Luenberger (N–L) measure of super-efficiency is developed based on a directional distance function. Although this N–L super-efficiency model does not suffer infeasibility problem as in the conventional radial super-efficiency DEA models, it can produce an infeasible solution in two special situations. The current paper proposes to modify the directional distance function by selecting proper feasible reference bundles so that the resulting N–L measure of super-efficiency is always feasible. As a result, our modified VRS super-efficiency model successfully addresses the infeasibility issues occurring either in conventional VRS models or the N–L super-efficiency model. Numerical examples are used to demonstrate our approach and compare results obtained from various super-efficiency measures.

1. Introduction

Data envelopment analysis (DEA) is a method for measuring relative efficiency of peer decision making units (DMUs). In recent years, DEA has been applied to various settings, such as performance evaluations in Olympic Games [1], estimating the importance of objectives in agricultural economics [2], regional R & D investment evaluations in China [3], and bankruptcy assessment for corporations [4]. In an effort to differentiate the performance of efficient DMUs, Andersen and Petersen [5] develop a super-efficiency model based upon the constant returns to scale (CRS) model [6]. However, when the concept of super-efficiency is applied to the variable returns to scale (VRS) model [7], the resulting model must be infeasible for certain DMUs [8].

Infeasibility restricts a wider use of super-efficiency DEA. Recent years have seen several studies addressing the infeasibility issue and the development of new super-efficiency models. For example, Lovell and Rouse [9] modify the conventional radial super-efficiency model by scaling up the concerning input vector (in an input-oriented case), or by scaling down the concerning output vector (in an output-oriented case). Also under the VRS assumption, Chen [10,11] replaces inefficient observations by their respective efficient projections, and performs super-efficiency analysis with this revised data set. Cook et al. [12] show that for infeasibility cases, one needs to adjust both the input and output levels to move an efficient DMU under evaluation onto the frontier formed by the remaining DMUs. They develop a two-stage process to address the infeasibility issue. Lee et al. [13] develop an alternative two-stage process to addressing infeasibility issue in the conventional VRS super-efficiency models. On the other hand, based on the directional distance function [14], Ray [15] develops a procedure to obtain Nerlove–Luenberger (N–L) measure of super-efficiency in a single model to adjust both input and output levels. As a result, this N–L super-efficiency model does not pose a similar infeasibility problem in the conventional VRS super-efficiency models.

However, Ray [15] points out that the N–L super-efficiency model fails in two special situations. First, no feasible solution exists if the zero input value is present in a DMU under evaluation and all other DMUs in the reference set are positive-valued in that input. Second, when an N–L super-efficiency score is greater than 2, the model will yield an efficient projection involving negative output quantities. In fact, zero data are problematic in any super-efficiency models. For example, Lee and Zhu [16] show that either the conventional VRS super-efficiency or the two-stage super-efficiency procedure in [12] will become infeasible when zero data are present. Therefore, it is necessary to address the two issues presented in [15].

The current paper shows that we can choose a proper reference input–output bundle in the directional distance function (DDF) [14], and modify Ray's DDF-based VRS super-efficiency model [15]. The new super-efficiency model successfully addresses the infeasibility issues occurring either in conventional VRS models or the N–L super-efficiency model.

The remainder of this paper is organized as follows. Section 2 introduces the DDF and its previous applications in N–L efficiency and super-efficiency assessments. Section 3 proposes a modified DDF, based on which a new VRS DEA model is developed for

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super-efficiency measurement. This new super-efficiency model addresses the infeasibility issues occurring either in conventional VRS models or the N–L super-efficiency model. Section 4 illustrates the new approach by using the data set from [8]. Section 5 concludes with a summary of our contributions.

2. Directional distance function and super-efficiency

Assume that there are n DMUs producing the same set of outputs at the cost of the same set of inputs. Unit j is represented by $DMU_j (j=1,...,n)$, whose ith input and rth output are denoted by $x_{ij} (i=1,...,m)$ and $y_{rj} (r=1,...,s)$, respectively. Then under the standard assumptions of convexity and free disposability of inputs and outputs, the production possibility set (PPS) formed from the above set of n DMUs is represented by

$$T=\left\{(x_i,y_r)\,|\,x_i\ge\sum_{j=1}^{n}\lambda_j x_{ij}, i=1,...,m;\ y_r\le\sum_{j=1}^{n}\lambda_j y_{rj}, r=1,...,s;\ \sum_{j=1}^{n}\lambda_j=1, \lambda_j\ge 0, j=1,...,n\right\} \tag{1}$$

Consider an input–output bundle (x_{io},y_{ro}) and a reference input–output bundle (g^x,g^y). Then based on the PPS (1), the directional distance function (DDF) is defined as [14]

$$D(x_{io},y_{ro};g^x,g^y)=\max\ \beta : (x_{io}+\beta g^x, y_{ro}+\beta g^y)\in T \tag{2}$$

The reference bundle (g^x,g^y) can be chosen in an arbitrary way, which makes the DDF varies with reference to any specific DMU. Chambers et al. [14] select $(-x_{io},y_{ro})$ for (g^x,g^y), and obtain the standard DDF as

$$D(x_{io},y_{ro})=\max\ \beta : ((1-\beta)x_{io},(1+\beta)y_{ro})\in T \tag{3}$$

In DDF (3), each input is decreased and each output is increased simultaneously by the same proportion β.

The DDF-based VRS model for efficiency with respect to PPS (1) is [14]

$$\begin{aligned}&\max\ \beta\\ &s.t.\sum_{j=1}^{n}\lambda_j x_{ij}\le(1-\beta)x_{io}, i=1,...,m\\ &\sum_{j=1}^{n}\lambda_j y_{rj}\ge(1+\beta)y_{ro}, r=1,...,s\\ &\sum_{j=1}^{n}\lambda_j=1, \lambda_j\ge 0, j=1,...,n\end{aligned} \tag{4}$$

As pointed out in [15], the optimal value of β is the Nerlove–Luenberger (N–L) measure of technical inefficiency for the evaluated DMU, whose efficiency can be calculated as $(1-\beta)$. Efficiency scores can be used to rank those technically inefficient DMUs, but fail to differentiate efficient DMUs. Thus super-efficiency, which implies the possible capability of a DMU in reducing its outputs or increasing its inputs without becoming inefficient, is also applied into directional distance function. For $DMU_k(x_{ik},y_{rk})$, the PPS for super-efficiency is modified as

$$T_k=\left\{(x_i,y_r)\,|\,x_i\ge\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j x_{ij}, i=1,...,m;\ y_r\le\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j y_{rj}, r=1,...,s;\ \sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j=1, \lambda_j\ge 0, j=1,...,n, j\ne k\right\} \tag{5}$$

Then the directional distance function for DMU_k concerning the new PPS (5) is

$$D_k(x_{ik},y_{rk})=\max\beta_k : ((1-\beta_k)x_{ik},(1+\beta_k)y_{rk})\in T_k \tag{6}$$

The related VRS model for calculating the N–L super-efficiency of DMU_k is developed as [15]

$$\begin{aligned}&\max\beta_k\\ &s.t.\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j x_{ij}\le(1-\beta_k)x_{ik}, i=1,...,m\\ &\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j y_{rj}\ge(1+\beta_k)y_{rk}, r=1,...,s\\ &\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j=1, \lambda_j\ge 0, j=1,...,n, j\ne k\end{aligned} \tag{7}$$

A negative optimal value of β_k indicates the same proportion to be scaled down for the output bundle while to be scaled up for the input bundle in order to get an attainable input–output mix in PPS (5). Smaller β_k indicates that DMU is more N–L super-efficient.

In general, model (7) is always feasible. However, Ray [15] points out two exceptions. One is that the unit under evaluation DMU_k has at least one input i_o at the zero level, and all other DMUs in the reference set are positive-valued in that input, i.e., $x_{i_ok}=0$ while $x_{i_oj}>0, j=1,...,n, j\ne k$. In such a case, DDF-based super-efficiency model (7) becomes infeasible because its first set of constraints cannot be satisfied.

The other case is one where for some input i_o, there is $2x_{i_ok}<\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j x_{i_oj}$ for all λ_js combinations satisfying $\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j=1$ and $\lambda_j\ge 0, j=1,...,n, j\ne k$. Thus β_k is restricted to a value lower than -1. This will result in a reference point with negative output values.

3. Modified DDF-based super-efficiency

In this section, we tackle the above two infeasibility problems in N–L super-efficiency model (7). Note that these infeasibility issues are caused by the choice of (g^x,g^y), which results in the same changing proportion β taken by each input (decreased) and output (increased) simultaneously. We can choose a different (g^x,g^y) so that the above infeasibility cases will not occur. In other words, we can consider a reference input–output bundle different from the conventional selection in [14], by using $(-ax_{io}-1, by_{ro}+1)$ for (g^x,g^y), and obtain a new directional distance function as

$$D(x_{io},y_{ro})=\max\ \beta : ((1-\beta a)x_{io}-\beta,(1+\beta b)y_{ro}+\beta)\in T \tag{8}$$

Here both a and b are pre-determined positive parameters. We will develop a procedure to decide for such parameters to address the two infeasibility issues.

For DMU_k, the super-efficiency model based upon (8) can be written as

$$\max\beta_k$$

$$s.t.\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j x_{ij}\le(1-\beta_k a)x_{ik}-\beta_k, i=1,...,m \tag{9.1}$$

$$\sum_{\substack{j=1\\ j\ne k}}^{n}\lambda_j y_{rj}\ge(1+\beta_k b)y_{rk}+\beta_k, r=1,...,s \tag{9.2}$$