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On the qualitative properties of the optimal income tax

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ABSTRACT

We explore the precise requirements for the qualitative results on optimum income taxation to hold, with the aim of extending their application to a larger space of solutions than that of continuous, piecewise differentiable functions assumed in the literature. In particular, properties (R1)–(R8) in Ebert (1992) are shown to hold when the endogenous variables of the problem are defined by non-smooth or even discontinuous functions, provided consumption is supposed to be normal and leisure non-inferior. Moreover, the referred properties continue to hold, without assuming the normality of consumption, if it is supposed that the function descriptive of gross income becomes absolutely continuous.

In addition, a characterization of the set of potential solutions stemming from Lebesgue's Decomposition Theorem has been used to analyze the relevance of properties (R1)–(R8), vis-à-vis other possible features of optimal tax schedules. The conclusion is that, even assuming the normality of consumption, the case for regressivity should be viewed, on the lines suggested by Kaneko (1982) within a somehow different model, as an exceptional outcome versus other income tax structures that may arise at the optimum.

1. Introduction

The theory of optimum income taxation has largely been based on numerical simulations of special cases, for different values of the relevant parameters intervening in the model [e.g. Mirrlees (1971) and Tuomala (1984)]. In contrast, only few conclusive results have been attained on the qualitative nature of the optimal tax without adopting particular specifications of the functions involved, i.e. the social welfare function, the utility function, the production function and the density function.

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Even so, much of the research on qualitative properties of the tax was undertaken under the so-called ''first-order approach'', a relaxed procedure that neglects in the analysis some crucial monotonicity constraint expressive of second-order conditions for utility maximization [e.g. Mirrlees (1971), Sadka (1976) and Seade (1977, 1982)]. Such a limitation has been overcome in subsequent contributions with the application of the more appropriate procedure of the ''second-order approach''. This has allowed, not only to confirm the rigorous validity of the qualitative properties previously obtained,[1] but also to achieve new insights on the sign and shape of optimal tax schedules. The trouble lies however in that, to cope with the referred monotonicity constraint, now explicitly considered in the analysis, authors customarily resort to the assumption that the function descriptive of gross income comes to be continuous and, to some extent, differentiable.[2]

Among other advantages, if the functions concerned are continuous and, say, piecewise smooth one can express their increasing monotonicity by imposing the constraint that their derivative is non-negative, and thus characterize the solution by relying on standard optimal control techniques. Nevertheless, the cost of such advantages is a loss of generality for disregarding the potential solutions that do not present the supposed continuity and smoothness properties.[3] This has been corroborated in Ruiz del Portal (2008) by showing that, in contrast with standard results optimal income taxes turn out to be nowhere regressive, or even progressive, on ranges where the function descriptive of gross income is non-constant but singular, namely, where its corresponding derivative vanishes almost everywhere.

The main goal of this paper is accordingly to relax the usual requirements in gross income trajectories for the qualitative results on optimal tax schedules to hold. To this end, we shall first derive, with the help of a theorem in control theory by Makowski and Neustadt (1974), the necessary conditions that prevail under the second-order approach, without invoking the continuity and smoothness assumptions adopted in the standard literature. Once this is done, we shall be in disposition, not only to confirm the referred qualitative results, but also to evaluate their relevance in the light of the special assumptions on gross income solutions needed for their derivation, as compared with those assumptions required for achieving optimal tax schedules of a different structure.

In particular, properties (R1)–(R8) in Ebert (1992) will be shown to hold when gross and after-tax income functions may fail to be smooth or even continuous, provided consumption is supposed to be normal and leisure non-inferior. Moreover, such properties (R1)–(R8) will prove to hold as well, without assuming the normality of consumption, if it is supposed alternatively that gross and after-tax income functions become absolutely continuous. Notwithstanding these results, however, we shall check from the functions appearing in the Lebesgue's decomposition of the solution that, contrarily to what accepted heretofore, the case for regressivity must be viewed as an exceptional possibility versus other tax structures that might arise at the optimum.

As to the choice of Makowski and Neustadt (1974, Theorem 12.1), we must remark that its setup contains, among other advantages, mixed control-state-type equality constraints which are continuously differentiable with respect to the state and control variables, but only measurable regarding the type. This allows describing, after some transformations, the monotonicity constraint through a non-decreasing function that is just absolutely continuous, therefore easy to handle under the methods of control theory.

In the next section we recollect the central features of the model of optimum income taxation, and demonstrate the validity of Ebert's (1992) necessary conditions under just two weak restrictions imposed on gross income trajectories. Section 3 discusses the conditions still required to obtain the qualitative results on optimal tax schedules both, when consumption is supposed to be a normal good, and when this assumption is dropped. Section 4 compares the different tax structures that emerge,

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1 Such results state, in essence: optimal tax rates lie between zero and one, dropping to zero at the top and bottom of the scale of incomes.

2 See Brito and Oakland (1977), Brunner (1993) and Ebert (1992).

3 The same was pointed out from the very beginning by Mirrlees (1971, Section 4), when warning that the differentiability of the variables implied in the derivation of an optimal income tax is at least doubtful.