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Default and bankruptcy in an entrepreneurial economy with incomplete markets

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ABSTRACT

We build a general equilibrium model with incomplete markets, production, default, and bankruptcy. The existence of equilibrium is proved. Theoretically, under appropriate conditions, we show that the reduced-form entrepreneurial equilibrium and profit-maximization entrepreneurial equilibrium, as defined by Magill and Quinzii (1996), are equivalent. In addition, we find an inverse relationship between the economy real interest rate and the probability of default. This result is empirically tested by applying the Cox proportional hazards model with time-dependent covariates for a sample of sole proprietorships' unsecured credit operations in the Brazilian economy. The estimates confirm the findings from the theoretical model.

1. Introduction

There is no doubt that the production process has a prominent role in the functioning of any modern economy. This process is carried out by business enterprisers which are classified according to the firm ownership structure and methods used to finance investments. Usually, a general classification is composed of sole proprietorships (individually owned firms), partnerships, and corporations. Magill and Quinzii (1996), for instance, report that in the middle 1990s the United States of America recorded 21.5 million business enterprisers, of which 16.1 million were sole proprietorships, 1.8 million partnerships, and 3.6 million corporations. For the tax year 2007, the summer 2009 edition of the IRS's Statistics of Income Bulletin featured about 23.1 million non-farm sole proprietorships, an increase of more than 40% over the Magill and Quinzii data. Unlike corporations, which can raise capital by issuing stock, small business entrepreneurs have access to limited credit channels. Thus, they are subject to standard credit market conditions, similar to ordinary consumers, to finance their investment decisions.

In this scenario, sole proprietorships are directly affected by monetary policy measures, which use the nominal interest rate to stabilize inflation and economic activity. They are affected in the same fashion as individual consumers by such policy, essentially through variations in the economy base interest rate and the loan interest rate. In addition, the credit channel of monetary policy, as stressed by Bernanke et al. (1996), might result in another effect. Lower interest rates implied by an expansionist monetary policy, for instance, may result in banks lending more money to borrowers that were regarded in the past as too risky. This strategy might raise the default rate of the economy. Recent empirical evidence provided by JimTnez et al. (2008), Ioannidou et al. (2008), Divino et al. (2010), and Batista et al. (2011) support this argument.

The aim of this paper is to develop a general equilibrium model with production and incomplete markets that incorporates an active credit market (with strictly positive trade) and allows for decisions of defaulting by consumers and bankrupting by sole proprietorships to emerge in equilibrium. First, the theoretical model is used to demonstrate that the inclusion of production, in the form of sole proprietorships, in a general equilibrium model with incomplete markets, is compatible with good orderly of the markets. Second, we build a data set for small entrepreneurs in the Brazilian economy and test the theoretical results on the relationship between the probability of default and interest rates.

The model is inspired in two major sources: the pioneering works by Dubey et al. (2005) and Magill and Quinzii (1996). The first model incorporates default in the basic Arrow–Debreu general equilibrium framework with incomplete markets and, to prevent default, introduces a direct penalty in the agents' utility. An

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important feature of this model is that default emerges in equilibrium, differently from other studies such as Alvarez and Jermann (2000) and Kehoe and Levine (1993, 2001), where there are endogenous solvency constraints but no default in equilibrium. Our second source, Magill and Quinzii (1996), develops a model with incomplete markets where agents are allowed to be entrepreneurs. They define two kinds of equilibrium, represented by reduced-form entrepreneurial equilibrium and profit-maximization entrepreneurial equilibrium, which are proved to be equivalent under certain conditions. A third kind of equilibrium, called extensive-form entrepreneurial equilibrium, is introduced in order to demonstrate the indeterminacy of financial policies which have no effect on the real allocations in equilibrium. These results hold under the strong condition that there is no default by consumers and no bankruptcy by firms.

Our model combines features of both Dubey et al. (2005) and Magill and Quinzii (1996) in the sense that agents, while consumers, can default and, as entrepreneurs, go bankrupt. The penalties are placed in the utility function in order to match our data set, where small entrepreneurs offer no collateral to back promises. Collateral is another mechanism used to enforce financial contracts, as pioneered by Geanakoplos and Zame (2007). Some empirical applications of this modeling strategy can be found in Fostel and Geanakoplos (2008).

Although our work only deals with sole proprietorships,[1] one must remember that there is a wide literature on general equilibrium with production and incomplete markets. Those works, however, are concentrated on partnerships, as in Radner (1972). For corporations, the major challenge is to determine the objective of the firm, which is not well defined in the presence of incomplete markets. An excellent survey on this issue can be found in Magill and Quinzii (1996). Contributions by Ekern and Wilson (1974), Ekern (1975), DrFze (1974), and Grossman and Hart (1979) also stand out. Recent advances in modeling production in general equilibrium with incomplete markets can be found in DrFze et al. (2008), Magill and Quinzii (2009), and Bisin et al. (2010)).

In the empirical analysis, we apply survival analysis to estimate the probability of default under the influence of both individual characteristics and time-dependent macroeconomic covariates. The idea is to test whether the relationship between the probability of default and interest rates derived by the theoretical model holds for Brazilian data. Following Cox (1972, 1975), we estimate the Cox proportional hazards model with time-varying covariates for a large sample of sole proprietorships from a major Brazilian bank. Especially in credit score models, that technique has been applied by several authors, including Lane et al. (1986), Banasik et al. (1999), Stepanova and Thomas (2001, 2002), Andreeva (2006), and Bellotti and Crook (2007). Among the alternative statistical procedures, the survival analysis based on the Cox (1972, 1975) proportional hazards model has been chosen because it matches the loan default process, allows modeling of both probability and time of default, provides forecast as a function of time, enables the inclusion of time-dependent covariates, and does not require any assumption on the probability distribution of the data.

Our major contribution is twofold. On the theoretical front, we incorporate default and bankruptcy in the entrepreneurial economy of Magill and Quinzii (1996) and derive a result on the separation of economic activities between consumers and producers. In addition, we offer conditions for a non-trivial equilibrium in the credit market, in the sense that there will always be positive trade, and demonstrate that the reduced-form entrepreneurial equilibrium and profit-maximization entrepreneurial equilibrium, as defined by Magill and Quinzii (1996), are equivalent. Finally, in equilibrium, we show that there is an inverse relationship between the probability of default and the economy base interest rate, and a direct relationship between the probability of default and the loan interest rate.

On the empirical side, the latter theoretical results are confronted by a large data set for sole proprietorships from a major Brazilian bank.[2] The estimation of the Cox proportional hazards model with time-varying covariates indicates that the theoretical signs on the relationship between the probability of default and the distinct interest rates are confirmed by the data. The intuition for this result is that reductions in the economy base real interest rate, as implied by an expansionist monetary policy, lead banks to easy credit history analysis and assume more credit risk. Banks will try to compensate for financial losses due to a lower base real interest rate by expanding credit operations. This strategy will bring borrowers with a higher probability of default to the financial market. On the other hand, a higher loan interest rate increases the probability of default because it reduces the borrower's capacity of debt repayment.

The paper is organized as follows. The next section describes the theoretical model. Section 3 discusses the equilibrium concepts used in the analysis. All theorems, corollaries and propositions establishing the main theoretical results are in Section 4. Section 5 presents the econometric model. The empirical results are reported and analyzed in Section 6. Lastly, Section 7 is dedicated to the concluding remarks.

2. The model

The model is a two-period general equilibrium model with production and financial markets, with time $t = 0,1$. Uncertainty is represented by a finite set $S = \{1,2,\ldots,S\}$ of states of nature to be revealed in the second period, $t = 1$. In the first period, $t = 0$, there is no uncertainty.

The economy has F firms and H agents. We assume that the number of agents, H, is greater than the number of firms, F. That is, $F \leqslant H$. Our analysis is specialized for the case in which each firm $f \in F$ has a sole owner $h \in H$. To formalize this requirement, we exogenously define the ownership map h: $F \to H$, which associates to each firm f its owner $h(f)$. It is also convenient to consider the function f: $H \to F$ which associates the firm $f(h)$ to each individual h. If $f(h) = \emptyset$, then the individual is not an entrepreneur.

We assume that there are financial markets.[3] In these markets J bonds are traded. These are characterized by their exogenous payoffs $r^j \in R^S_+, j \in J$, a penalty for defaulting and/or going bankrupt $\lambda \in R^{JS}_{++}$, and limits on short sales $v \in R^J_+$. Thus, the financial structure is defined by the payoff matrix: $R = [r^1 \cdots r^J]_{S\times J}$. Let us denote by $r_s \in R^J_+$ the s-line of the payoff matrix R which represents the payoff of the J bonds in each state of nature for each unit sold. There is only one good in each period and in each state of nature so that the commodity space is R^{S+1}.

Remark 2.1. Bonds available can be interpreted as being credit products for both consumers and entrepreneurs.

All agents in the economy are consumers who may also be entrepreneurs. Each agent $h \in H = \{1,\ldots,H\}$ is characterized by a utility function $U^h : R^{S+1}_+ \to R$ and an endowment vector of commodities $\omega^h \in R^{S+1}_{++}$. Each entrepreneur $h(f)$ of the firm f is characterized by the production set $Y^{f(h)} \subset R^{S+1}$ which summarizes all investment projects available.

Thus, an entrepreneurial economy with default and bankruptcy is a vector $\mathcal{E} = [(u^h, w^h)_{h\in H}; (Y^f)_{f\in F}; (R, \lambda^h, v)]$.

[1] Current version of Fisher's (1930) seminal work.

[2] The name of the bank will not be revealed for confidential reasons.

[3] Financial markets are available for both consumers and producers in the first period.