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Controlling endogenous fluctuations of human capital investment in an OLG economy

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ABSTRACT

A bifurcation occurs when there is a sudden qualitative or topological change in the behavior of the original system by varying one or more parameters (the bifurcation parameters) of the original system. Bifurcation can cause unacceptable new conditions or instability in the economy system. Its control is done by designing a controller input, thereby achieving desirable dynamical behavior. This paper deals with the control of a bifurcation caused by a rise in information costs in a human capital investment model. By employing the delayed feedback control (DFC) method, unstable fluctuations stemming from the system can be controlled without changing its original properties. In addition, we show the effectiveness and feasibility of the developed methods in the system with explicit functions.

1. Introduction

Since Andronov et al. first studied bifurcation control in 1960s, bifurcation control has been intensively studied in both science and social science fields. Different kinds of bifurcations may take place in a nonlinear system. Magnus (2009) discusses saddle-node (fold) bifurcation in preloaded spherical roller thrust bearings systems. Galluzzo et al. (2008) describe transcritical bifurcation in a nonlinear bio-reactor system. Wei and Zhang (2008) consider a pitchfork bifurcation in a class of n-dimensional neural network model with multi-delay. Flip (period-doubling) bifurcation occurs in a nonlinear bio-reactor system (Griselda and Jorge, 2006). Ding et al. (2009) study Hopf bifurcation in an Internet congestion control algorithm of TCP/AQM networks. Anna (2006) deals with Neimark (secondary Hopf) bifurcation in the duopoly model.

Complicated dynamics can exist in economic models. Empirically, enrollment rates in higher education vary significantly over time. Michael and Jan (2003) illustrate a flip bifurcation economy based on a human capital investment OLG model. A flip bifurcation occurs in a dynamical system which switches to a different dynamics with twice the period of the original system.

Unstable behavior in an economic system is often an unfavorable outcome of bifurcation in traditional economics. This holds true in the demand for education. Shortages in qualified labor may slow economic growth. Also, college and university may costly excess capacity. An individual may regret her educational investment decision.

Economic efficiency may be increased by stabilizing unstable fluctuations. As emphasized by Chen et al. (2000), bifurcation control not only is important in its own right, but also suggests a viable and effective way for chaos control. This is because bifurcation and chaos are usually ‘twins’, in particular, flip bifurcation results in chaos in many nonlinear dynamical systems. There are many algorithms for bifurcation control (Ott et al., 1990; Chen et al., 1998, 2000; Lan and Li, 2008), but suitable methods for bifurcation control in economics are limited. One strand of the economic literature uses linear controllers, such as in Matsumoto (2006). Another strand of the literature uses nonlinear controllers, such as the OGY method (Ott et al., 1990) in Diana and Vivaldo (2005), the DFC method in Chen and Chen (2007), Tan et al. (2008), Hassan and Aria (2008, 2009).

In this paper, the DFC method is applied to a human capital investment OLG model set up by Michael and Jan (2003). We show how flip bifurcation can be controlled via the delayed feedback control (DFC) method in the human capital investment OLG model. In addition, we discuss the effectiveness and feasibility of the developed methods by example with explicit functions.

The paper is organized as follows. In Section 2, the basic nonlinear economic model is given in which flip bifurcation exists. Section 3 is devoted to controlling fluctuations by using the delayed feedback control method. Section 4 reports the results and suggests applications. Finally, concluding remarks are presented in Section 5.

2. The basic model

In this section, we will be studying the basic model which extends the work by Michael and Jan (2003).

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There are two sectors in the economy, sector H and sector L. The labor employed in section H is high-skilled. While the labor employed in section L is low-skilled. Each section produces the same commodity. As demand for low-skilled labor is perfectly elastic, the wage in sector L is normalized to be 1 unit of the production good. The production technology in section H is assumed to satisfy a concave function $f(l)$. The demand function for high-skilled labor $l^{\mathrm{d}}(w_t)$ is the solution to $f'(l_t)=w_t$, where w_t denotes the real wage rate for high-skilled labor in period t.

In each period t, a continuum of agents of mass one is born that lives for two periods. Agents have one time unit in each period. In the first period of their life, they choose investment options. If an agent does not invest in education, she will work in sector L all of her life. Her consumptions levels will be:

$$c_t = c_{t+1} = 1. \tag{1}$$

Otherwise, assume agent i invests private effort e_i in education. Then her consumption levels will be:

$$c_t = 1 - e_i, \tag{2}$$

$$c_{t+1} = w_{t+1}. \tag{3}$$

The agent decides to invest if and only if she expects lifetime utility to be at least as large as if she had not:

$$U(1 - e_i, w^e_{t+1}) \geq U(1,1). \tag{4}$$

Here, $U(c_t, c_{t+1})$ is the lifetime utility function.

By equating these utility levels we denote the marginal effort level by $e^*_t = e(w^e_{t+1})$. And the effort costs $e(w^e_{t+1})$ are distributed according to the distribution function $F(e(w^e_{t+1})$. There are K different kinds of predictors $w_{k,t+1}$, $k=1,2,\ldots, K$. Every agent uses one of the predictors, n_{kt} is the fraction using predictor $w_{k,t+1}$. Thus the total supply of high-skilled labor in $t+1$ is:

$$l^s\left(w_{1,t+1},\cdots,w_{K,t+1}\right) = \sum_{k=1}^{K} F\left(e\left(w_{k,t+1}\right)\right). \tag{5}$$

Combining Eq. (5) with the labor market clearing condition, we have:

$$l^s\left(w_{1,t+1},\cdots,w_{K,t+1}\right) = l^{\mathrm{d}}\left(w_{1,t+1},\cdots,w_{K,t+1}\right). \tag{6}$$

The newborn generation makes an investment decision based on information from inter-generational spill-overs:

$$n_{k,t+1} = H_\beta\left(R\left(w_{1,t+1},w_{t+1}\right) + C_1,\cdots,R\left(w_{K,t+1},w_{t+1}\right) + C_K\right). \tag{7}$$

Where $R(w_{k,t+1})$, w_{t+1} is regret for using each predictor k:

$$R\left(w_{k,t+1},w_{t+1}\right) = \int_{e(w_{t+1})}^{e(w_{k,t+1})} R(e,w_{t+1})\mathrm{d}F(e) \tag{8}$$
$$= U(1,1) - U(1-e_i,w_{t+1}),$$

C_k denotes information costs in using predictor k.

We assume agents make their decisions between rational expectations and naive expectation. Rational expectations imply $w^e_{t+1}=w_{t+1}$. Naive expectations imply $w^e_{t+1}=w_t \cdot n_t$ is the fraction of rational forecasters. $1-n_t$ is the fraction of naive forecasters. The labor market clearing condition can be rewritten as:

$$l^{\mathrm{d}}(w_{t+1}) = n_t F(e(w_{t+1})) + (1-n_t)F(e(w_t)). \tag{9}$$

Combining the implicit function theorem, the above equation implies there is a general function:

$$w_{t+1} = G(w_t, n_t). \tag{10}$$

Now consider a special case for the fraction of rational expectations:

$$n_{k,t+1} = H(\beta(R(w_t, G(w_t,n_t)) - C_r)). \tag{11}$$

We make use of the discrete choice model (see Michael and Jan, 2003).

Then the full model can be described as the following System I:

$$w_{t+1} = G(w_t, n_t), \tag{12}$$

$$n_{k,t+1} = H(\beta(R(w_t, G(w_t,n_t)) - C_r)). \tag{13}$$

The following propositions are proved by Michael and Jan (2003). We rewrite them here for convenience.

Proposition 2.1. *The steady state of the dynamic System I (Eqs. (12) and (13)) is: $(w^*, n^*)=(w^*, H(-\beta C_r))$. This steady state is locally stable if $|l^{\mathrm{d}}_w| \geq l^{\mathrm{d}}_w$. If $|l^{\mathrm{d}}_w| < l^{\mathrm{d}}_w$, there exists a critical value $(\beta C_r)^*$ of βC_r such that for $\beta C_r < (\beta C_r)^*$ the steady state is locally stable and for $\beta C_r > (\beta C_r)^*$ the steady state is unstable. Furthermore, $(\beta C_r)^*$ is implicitly given by: $H\left(-(\beta C_r)^*\right) = \frac{l^s_w - |l^{\mathrm{d}}_w|}{2 l^s_w}$. Under certain regularity conditions a flip bifurcation occurs for $\beta C_r = (\beta C_r)^*$. Then for βC_r close to $(\beta C_r)^*$ a two-period cycle exists. This cycle merges with the steady state at $\beta C_r = (\beta C_r)^*$.*

This proposition shows that endogenous fluctuation can occur if the value of βC_r is sufficiently high relative to the critical value $(\beta C_r)^*$. The emergence of such fluctuations can be explained as follows: if the system has a high value of β, agents respond quickly to differences in costs. If the system has a high value of C_r, there is high information cost in predicting. Both of these two factors have negative impact on the fraction of rational forecasters in the next generation. So the agents have more difficulty in making a rational decision. Consequently, there arise fluctuations between labor supply and demand which is an "unstable cobweb" dynamics. Hence, the wages will undergo unstable fluctuations. And these fluctuations in wage will increase aggregate regret in the following period. As a result, the same fluctuations appear in the next period, which gives rise to the emergence of an endogenous fluctuation cycle in this human capital model.

Consider the following explicit functions: The utility function is $U(c_t, c_{t+1}) = c_t^\gamma c_{t+1}^{1-\gamma}$. The production technology function in sector H is $f(l)=(\alpha/\mu)l^\mu$. We take the restriction $(1-\mu)(1-\gamma)=\gamma$, and use $x_t = w_t^{-(1-\gamma)/\gamma}$ and $\delta=(1-\gamma)/\gamma$. Aggregate regret can be computed as:

$$R(e_i, w_{t+1}) = 1 - (1-e_i)^\gamma w_{t+1}^{1-\gamma}, \tag{14}$$

$$R(x_t, w_{t+1}) = (x_{t+1} - x_t) - \frac{1+\delta}{2+\delta} x_{t+1}^{\frac{2+\delta}{1+\delta}} - x_t^{\frac{2+\delta}{1+\delta}}. \tag{15}$$

The full model can be written as:

$$x_{t+1} = P(x_t, n_t) = \left(G\left(x_t^{-\frac{1}{\delta}}, n_t\right)\right)^{-\delta} = \frac{1-(1-n_t)x_t}{\alpha^\delta + n_t}, \tag{16}$$

$$n_{t+1} = Q(x_t, n_t) = H\left(x_t^{-\frac{1}{\delta}}, n_t\right) = \frac{1}{1 + \exp[-\beta(R(P(x_t,n_t),x_t) - C_r)]}. \tag{17}$$

The following proposition discusses about the local dynamic behavior of System II (Eqs. (16) and (17)).