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Risk capital allocation for RORAC optimization

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ABSTRACT

This paper considers the financial optimization problem of a firm with several sub-businesses striving for its optimal RORAC. An insightful example shows that the implementation of classical gradient capital allocation can be suboptimal if division managers are allowed to venture into all business whose marginal RORAC exceeds the firm's RORAC. The marginal RORAC requirements are refined by adding a risk correction term that takes into account the interdependencies of the risks of different lines of business. It is shown that under certain stationarity conditions this approach can guarantee that the optimal RORAC will eventually be achieved.

1. Introduction

The allocation of risk capital in financial firms for the purpose of performance measurement and risk-return optimization is well established in theory as well as in practice. Throughout this paper, we will use the terms risk capital and economic capital synonymously for an estimate of the amount of equity a firm needs to cover potential losses generated by its business. In contrast to regulatory capital, which is calculated using externally given rules and methodologies, economic capital represents an internal estimate of the risks. While the use of economic capital and its decomposition into a sum of single contributions of sub-businesses has become a standard approach in many banks (see Rosen and Saunders, 2010) and insurance companies (see Myers and Read, 2001), the academic world is still discussing methodological aspects and, to an extent, even the very significance of this concept.

There are several strands of literature which deal with risk capital allocation from various points of view. Most articles can be attributed to the mathematical finance context, in which rigorous arguments and axiomatics form the main focus (e.g. Denault, 2001; Kalkbrener, 2005; Tasche, 2004; Buch and Dorfleitner, 2008). Another strand of literature has a definite insurance-linked perspective (e.g. Dhaene et al., 2003; Furman and Zitikis, 2008; Gatzert and Schmeiser, 2008) and seeks to explore the advantages of risk capital allocation for insurance companies. A third strand looks at risk capital allocation from a more financial economics point of view (e.g. Merton and Perold, 1993; Stoughton and Zechner, 2007) and is therefore more closely related to the question concerning why capital allocation is a sensible procedure from an economic perspective.

In any case, a sound risk capital allocation framework requires at least two theoretical fundaments, namely a proper definition of a risk measure and an allocation principle. The combination of these two items yields a concrete allocation rule. In addition, several ad hoc allocation rules, like e.g. the covariance allocation rule,[1] exist without explicit reference to the combination of a risk measure and an allocation principle. Much attention has recently been

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[1] See e.g. Kalkbrener (2005), who also points out the shortcomings of this allocation rule. Urban et al. (2004) use the covariance principle for calculating relative weights of each segment independently of the overall portfolio risk measure.

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given to coherent risk measures (Artzner et al., 1999), which have several economically favorable properties, and to the gradient allocation principle (Tasche, 2008; Rosen and Saunders, 2010), also sometimes called the Euler allocation principle. The gradient allocation principle is well-suited to firms with homogeneous sub-businesses consisting of a continuum of single contracts, whereas in the case of few large single contracts an incremental allocation (Merton and Perold, 1993) seems to be more appropriate, where the risk capital allocated to sub-businesses is derived from looking at the firm with and without the sub-business under consideration and allocating economic capital proportional to the difference in overall risk capital.

While many contributions examine technical aspects of risk capital allocation rigorously and in great detail, the actual economic justification remains mostly verbal. Typically, it is stated that the allocation is necessary to control risks ex ante by assigning limits to individual business units and its necessity for performance measurement is emphasized. On the other hand, risk capital allocation is also subject to criticism. In fact, Gründl and Schmeiser (2007) argue that capital allocation is completely senseless and that firms should rather refrain from using it. Even if one does not wish to follow this argument, the question emerges concerning why the optimum amounts of every line of business are not more adequately directly optimized by the headquarters.

The contribution of Stoughton and Zechner (2007) is the first to actually consider an economic optimization problem. The authors show that if the firm as a whole pursues maximization of the economic value added it is consistent with allocating capital to the sub-businesses, which are characterized by private information of managers, and allowing them to maximize the economic value added, based on the allocated capital. However, due to the restriction to normally distributed risks and a very specific incremental Value-at-Risk allocation rule, which is largely identical to the covariance allocation, their results are only of limited usefulness in terms of practical application.

This paper focuses on financial firms with different lines of business, for which the managerial decision concerns whether to expand or reduce rather than to create newly or abandon completely. We do not restrict ourselves to certain specific risk measures or distributional assumptions. Our approach comprises banks and insurance companies, both of which are subject to risk capital allocation. In banks the economic capital to be allocated could cover market, credit, and operational risk (Alessandri and Drehmann, 2010; Breuer et al., 2010; Embrechts et al., 2003) or classically credit risk in a portfolio context (Rosen and Saunders, 2010), while in insurance companies risk capital could be allocated for different lines of insurance contracts (Urban et al., 2004).

This paper contributes to the literature by developing a justification of risk capital allocation with a rather mathematical finance argumentation, which is well-suited to the many extensively axiomatic contributions made on risk measures and economic capital found in the literature. To our knowledge there is no contribution, which argues without restricting the probability distribution of losses and the risk measure chosen that capital allocation could be reasonable when pursuing a maximization problem. In this paper we fill the gap by developing a procedure concerning capital allocation that is designed to maximize the RORAC of a company. Our analysis is based on the work of Tasche (2004) who, however, is not able to state a maximization problem due to assumptions, which are too simplistic. We assume that the segment managers have superior knowledge concerning the possible profits induced by segment reductions or expansion, while the risk of the portfolio is calculated centrally by the headquarters. Based on this we question RORAC maximization utilizing naive risk capital allocation and develop a more sophisticated rule for RORAC maximization.

The remainder of this paper is structured as follows: firstly, we present the general organizational framework for capital allocation and return maximization in Section 2. Afterwards, we introduce our specific model in Section 3 and derive certain requirements for the existence of a company-wide optimal RORAC. Moreover, we propose an explicit control strategy that directs a firm to the optimal RORAC. We then give a numerical example in Section 4, showing how a classical risk allocation rule to sub-businesses can impede a company in attaining its optimal RORAC and elaborating the success of the afore-mentioned control strategy. Practical aspects are discussed in Sections 5 and 6 concludes our paper.

Notice that the proofs of all lemmas, theorems, and corollaries can be found in the appendix.

2. General organizational framework

We consider a firm with n lines of business (subsequently called segments), in which each segment conducts a certain amount of business. Let the vector $u = (u_1, \ldots, u_n) \in \mathbb{R}^n$ symbolize the amount of business of each segment. We call u the *portfolio* and $U \subseteq \mathbb{R}^n$ the set of all portfolios. Let the future profits of segment $k = 1, \ldots, n$ stemming from an amount of business u_k be represented by the discrete-time process $(Y_k^t(u_k))_{t=1,2,\ldots}$ of integrable random variables on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with natural filtration $(\mathcal{F}^t)_{t\geqslant 0}$. To be precise, $Y_k^t(u_k)$ denotes the profits generated by segment k in the period $(t-1, t)$. Note that the portfolio u can change over time. Since the firm's overall profit consists of the profits of each segment, we can add up $Y^t(u) := \sum_{k=1}^n Y_k^t(u_k)$, so that Y^t denotes the total profit of the firm in the corresponding period.

The risk capital of a portfolio u for time interval (s,h) is assumed to be $\varrho((Y^t(u))_{t=s+1,\ldots,h})$. Formally, ϱ is a multi-period risk measure, i.e., a mapping of the set of all tuples of random variables $(Y^{s+1}(u),\ldots,Y^h(u))$ into the real numbers $\mathbb{R}$. For further details on the construction of such a multi-period risk measure, see e.g. Artzner et al. (2007), Frittelli and Scandolo (2006). If ϱ is sub-additive, the firm's overall risk $\varrho((Y^t(u))_{t=s+1,\ldots,h})$ will be lower than the sum of the segments' stand-alone risks $\sum_{k=1}^n \varrho\left((Y_k^t(u_k))_{t=s+1,\ldots,h}\right)$. This motivates the definition of an allocation principle, which fully proportions the firm's overall risk to the individual segments.

Definition 2.1. Let $\mathcal{P}$ be defined as a set of tuples $(Y^{s+1},\ldots,Y^h,u)$, with $u \in U$, where U is a non-empty set in $\mathbb{R}^n$ and $s < h$. Given a risk measure ϱ, an *allocation principle* on $\mathcal{P}$ is defined as a mapping $A^\varrho : \mathcal{P} \to \mathbb{R}^n$ with

$$A^\varrho : (Y^{s+1},\ldots,Y^h,u) \mapsto \begin{bmatrix} A_1^\varrho(Y^{s+1},\ldots,Y^h,u) \\ \vdots \\ A_n^\varrho(Y^{s+1},\ldots,Y^h,u) \end{bmatrix}$$

such that

$$\sum_{k=1}^n A_k^\varrho(Y^{s+1},\ldots,Y^h,u) = \varrho((Y^t(u))_{t=s+1,\ldots,h}).$$

The expression $A_k^\varrho(Y^{s+1},\ldots,Y^h,u)/u_k =: a_k^{(s,h)}(u)$ is referred to as the *per-unit risk contribution* of segment k for time interval (s,h).

The capital allocation can be seen as a means to split up the diversification benefits stemming from the pooling of the segments' risks. It should be noted that the risk capital allocated does not coincide with real capital invested to fund the business in the segments. Since we consider financial firms, we can assume that the investments are financed to a large extend through debt, while equity in the form of economic capital is only essential to cover the risks of the investments. Therefore, risk capital allocation in a financial institution requires a different approach than classical