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Core and equilibria in coalitional asymmetric information economies☆

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ABSTRACT

We study coalitional economies under uncertainty and asymmetric information, assuming a finitely additive measure space of agents and finitely many possible states of nature. We introduce a suitable core notion showing that it is equivalent to Walrasian expectations equilibria. The finitely additive approach proposed in the paper permits also asymmetric information economy with countably many agents, rather than requiring only a continuum. Moreover, it allows us to overcome well-known criticisms related to the interpretation of individual private core notions.

1. Introduction

Radner (1968) introduced a model of an exchange economy with asymmetric information, namely an economy in which different economic agents may have different information. In this model every agent is characterized by a state-dependent utility function, a random initial endowment, an information partition and a prior belief. The trade is a reallocation of commodities and the contracts of trading are arranged before agents obtain any information about the realized state of nature. The notion of Walrasian expectations equilibrium (or Radner competitive equilibrium) extends the Arrow-Debreu competitive equilibrium to this model: each agent maximizes his ex-ante expected utility subject to budget constraints. The information plays an important role in the definition of competitive equilibrium. Indeed, feasible net trades of each trader are required to be measurable with respect to his private information. Consequently, the information may enlarge the budget set of traders whenever it increases.

Dealing with cooperative characterization of Walrasian expectations equilibria, several alternative notions of core are possible. Mainly they depend upon the initial information and the opportunities of communication that the members of a blocking coalition have.

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In Yannelis (1991), the notion of private core is introduced based on the assumption that agents have no access to communication systems. Even in this context, blocking allocations for a given coalition are required to be measurable with respect to private information. It has been shown that the private core has many interesting properties: it is nonempty under standard continuity and concavity assumptions, it is coalitionally incentive compatible and rewards the information superiority of traders. Moreover, classical equivalence theorems with the set of Walrasian expectations allocations can be restored as proved in Einy et al. (2001), Graziano and Meo (2005), Bimonte and Graziano (2009), Hervés-Beloso et al. (2004, 2005). Results in Einy et al. (2001), Bimonte and Graziano (2009) and Hervés-Beloso et al. (2005) are proved for finite dimensional economies, those in Graziano and Meo (2005) and Hervés-Beloso et al. (2004) are valid when the commodity space is infinite dimensional, separable and has a positive cone with nonempty interior. In particular, as proved in Tourky and Yannelis (2001), the separability assumption is indispensable to get equivalence results even in the case of complete information economies. Moreover, all the cited papers deal with a nonatomic or with a finite space of agents.

In Vind (1964) a coalitional representation of a complete information economy is given and an equivalence result is provided in this new framework. The key idea is to replace individuals by coalitions and resource allocations by countably additive measures. Hence, in a coalitional model, primitive entities are coalitions each one characterized by a preference relation and an initial endowment.

Subsequently, in Armstrong and Richter (1984) the Core-Walras equivalence is studied in coalitional models dropping the assumption of countable additivity. The step from countable to finite additivity is motivated by the following substantial questions that we report from Armstrong and Richter (1984): "Why should a countable union of coalitions necessarily be a coalition? Why should we assume there are uncountably many people in the society? Can't we model our economy on a countable set such as the integers?".

It is worthwhile observing that the coalitional model generalizes the classical Aumann's model of a nonatomic economy (see Armstrong and Richter, 1984; Debreu, 1967). Going further, in the infinite dimensional setting the coalitional representation offers the advantage of furnishing the core equivalence under relatively plain assumptions. Indeed in this case, contrary to the individualistic model, it is not necessary to assume the separability of the commodity space (compare Basile and Graziano, 2001, Remark 4.1).

The aim of this paper is to consider asymmetric information economies from the coalitional point of view (we refer to Angeloni and Martins-Da-Rocha, 2009; Einy et al., 2001; Sun and Yannelis, 2007, for individualistic models of large exchange economies with asymmetric information). Following the same approach of Armstrong and Richter (1984) and Basile (1993), we assume that the primitive entities of the economy are coalitions, each one characterized by a preference relation, a random initial endowment and an information partition. As for individualistic models, in the definition of Walrasian expectations equilibria and core allocations the information has a central role. In a Walrasian expectations equilibrium each coalition maximizes its preference relation over its budget set where informational constraints are added to the classical ones.

Private core allocations in the coalitional framework are finitely additive vector measures that cannot be privately blocked. In the corresponding private blocking mechanism, the relevant allocations are required to be measurable with respect to the information of coalitions. Hence private core allocations are proved to be equivalent to Walrasian expectations allocations.

We notice that, until now, the Core and Walrasian expectations equilibria have been studied in nonatomic economies via countably additive measure theory. According to the Aumann classical contribution, the underlying Boolean algebra of coalitions was a σ-algebra (typically, the Lebesgue measurable subsets of the unit interval $[0, 1]$), the allocations were countably additive. No equivalence result has been found in any model approaching the generality of the finitely additive one. It is the aim of our paper to fill this gap, proving that these artificial technical assumptions can be removed even in the presence of uncertainty and asymmetric information. The first reason to eliminate unnecessary assumptions is that it makes the theory clearer. Secondly, real world applications involve only finitely many agents, hence finitely many coalitions. Thus, it is natural to assume that only finite unions and intersections can be observed, and consequently only finite additivity. There are several contexts in which the assumption of countable additivity is unnatural or unverifiable. For example, the finitely additive approach of the paper allows us to tackle the case of a countable set of asymmetrically informed agents. This case arises in the setting of limit economies. Since large economies with pure competition are a kind of limit of finite economies, it would be natural if they had countably many agents. Moreover, all the coalitions should be allowed. As there are no-Lebesgue measurable sets on the unit interval $[0, 1]$, a priori not all coalitions are allowed in the continuum. By means of finitely additive atomless economies with countably many agents this points of criticism can be avoided (observe that there are no countably additive, atomless and nontrivial measures on $(\mathbb{N}, 2^{\mathbb{N}})$). Economies that are only finitely additive arise also naturally looking at limits of sequences of finite economies where agent's characteristics are becoming very dispersed rather than clustered (see Armstrong, 1985). They have been found very useful in models of environments with a large number of agents and stochastic characteristics in connection with the failure of the law of large numbers in the continuum (compare Al-Najjar, 2004).

Finally we remark that, as a further advantage of our approach, the definition of private core introduced in the paper, allows to overcome a well known criticism related to the interpretation of individual private core notions. Indeed, as written in Forges et al. (2002), "... the very notion of the core is based on agents making agreements to trade among themselves, not through an anonymous market. This clearly involves communication among agents, and it is then unreasonable to impose the restriction that an agent cannot entertain a contract which varies with information he does not possess." Since communication is necessary only at the aggregate level of non-null coalitions, it turns out that the coalitional framework is the natural setup to develop a core theory when information is asymmetric. Indeed, it allows to define the private information