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Portfolio management under epistemic uncertainty using stochastic dominance and information-gap theory

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Abstract

Portfolio management in finance is more than a mathematical problem of optimizing performance under risk constraints. A critical factor in practical portfolio problems is severe uncertainty – ignorance – due to model uncertainty. In this paper, we show how to find the best portfolios by adapting the standard risk-return criterion for portfolio selection to the case of severe uncertainty, such as might result from limited available data. This original approach is based on the combination of two commonly conflicting portfolio investment goals:

(1) Obtaining high expected portfolio return, and
(2) controlling risk.

The two goals conflict if a portfolio has both higher expected return and higher risk than competing portfolio(s). They can also conflict if a reference curve characterizing a minimally tolerable portfolio is difficult to beat.

To find the best portfolio in this situation, we first generate a *set* of optimal portfolios. This set is populated according to a standard mean-risk approach. Then we search the set using stochastic dominance (SSD) and Information-Gap Theory to identify the preferred one. This approach permits analysis of the problem even under severe uncertainty, a situation that we address because it occurs often, yet needs new advances to solve. SSD is attracting attention in the portfolio analysis community because any rational, risk-averse investor will prefer portfolio y_1 to portfolio y_2 if y_1 has SSD over y_2. The player's utility function is not relevant to this preference as long as it is risk averse, which most investors are.

Keywords: Epistemic uncertainty; Finance; Imprecise probabilities; Info-gap; Information-gap; Portfolio; Probability boxes; Stochastic dominance

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1. Introduction

A portfolio consists of a set of *segments*, each of which is predefined as a particular asset category, such as stocks, bonds, commodities, etc. Solving the selection problem means determining the best proportion each segment should be of the total investment. The portfolio selection problem is the subject of a vast body of work. The process can be divided into two phases. The first is asset allocation, in which investor philosophy, including risk position, is used to choose the best percentage of the portfolio to place in each segment. The second, rebalancing, responds to changes in asset values by adjusting the percentages so that the portfolio continues to accurately reflect the investment philosophy. This work focuses on allocation. The well known CAPM leads to various allocation strategies, including for example BIRR and BARRA (search the Web for further information about these). The correct treatment of the risk-reward problem addressed by Markowitz ([12]) is fundamental to such modern methods, and its extension to problems characterized by severe uncertainty motivates this report.

Little has been done to determine portfolio allocation when dependency relationships, such as correlations, among portfolio segment return distributions are unknown. We address this problem with a novel application of Information-Gap Theory (Ben-Haim [2]), using it together with the concept of *second-order stochastic dominance* (SSD) to help choose among portfolio allocations. SSD holds between two distributions r_1 and r_2 when the curves of their integrals do not cross. The slower rising curve is then said to have second-order stochastic dominance over the other curve. If r_2 has SSD over r_1, then we write $r_2 \succ_2 r_1$. Analogously, FSD (first-order stochastic dominance, $\prec_1$) applies if the distribution curves themselves do not cross. However, most investors are risk averse, and if $r_2 \succ_2 r_1$ then any risk averse player will prefer r_2 (e.g. Perny et al. [13]). FSD is thus an unnecessarily strong (and therefore undesirable) constraint for the risk averse player.

We build on a standard approach to finding optimal portfolios based on mean and risk and parameterized by amount of risk aversion, arising originally from Markowitz ([12]). The mean is the expectation (i.e. average) for a distribution describing the investment return, while the risk expresses the danger of loss or low returns and is typically a measure of the spread of the return distribution. Optimal portfolios are identified by finding the weights of the portfolio segments such that a mean-risk objective function is maximized (e.g. Elton et al. [8]). "Mean-risk" refers to a tradeoff between seeking a return distribution with a high mean, which is good, while minimizing the higher risk that tends to be associated with a high mean return, which is bad.

Formally, the problem to be considered is to find such a portfolio given the constraints

$$r = \sum_{i=1}^{s} w_i r_i \succ_2 \widetilde{R} \tag{1}$$

$$\sum_{i=1}^{s} w_i = 1 \tag{2}$$

where r is a portfolio return distribution, i is one of the s segments in the portfolio, w_i is the weight of segment i, and r_i is the return distribution of segment i. $\widetilde{R}$ is a given reference curve representing a minimally tolerable bound (the "risk limit") that the return distribution should not cross. As an additional constraint set (Eq. (2)), segment weights sum to 1 because each weight is a proportion of the whole. Each weight may be required to be within some interval in order to enforce a balance across segments, as might be specified by a company's business model constraints and investment policies.

In current practice optimization would typically be done without the $\succ_2$ constraint, but perhaps with other constraints present such as Value-at-Risk (VaR), which is known to be mathematically inconsistent, or Conditional VaR (CVAR) which is less intuitive but without VaR's consistency problem [3].

A given portfolio's return distribution can be tested for compliance with an SSD constraint using numerical integration. Numerical integration can be done straightforwardly by summing areas of trapezoids under the curve. The size and number of trapezoids to sum is determined by the step size chosen for the integration process. Given a step size, SSD is considered to hold if the summed areas of all trapezoids to the left of any given value x_i are lower for the return distribution r of a candidate portfolio than for reference curve $\widetilde{R}$. A set of representative return values $x_1, x_2, x_3, \ldots, x_n$ that are possible sample value of r should be checked. These points should cover low and high values of return, as well as a reasonable number of intermediate points (e.g. $m = 10$ or 20).