Journal of Economic Dynamics & Control 48 (2014) 1–25

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Portfolio management with robustness in both prediction and decision: A mixture model based learning approach

Shushang Zhu [a], Minjie Fan [b], Duan Li [c,*]

[a] Department of Finance and Investment, Sun Yat-Sen Business School, Sun Yat-Sen University, Guangzhou 510275, PR China
[b] Department of Statistics, University of California, Davis, One Shields Avenue, Davis, CA 95616, USA
[c] Department of Systems Engineering and Engineering Management, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

ARTICLE INFO

Article history:
Received 21 January 2014
Received in revised form
6 August 2014
Accepted 15 August 2014
Available online 23 August 2014

JEL classification:
C11
C61
G11

Keywords:
Portfolio selection
Mixture model
Robust optimization
Bayesian learning
Conditional value-at-risk

ABSTRACT

We develop in this paper a novel portfolio selection framework with a feature of double robustness in both return distribution modeling and portfolio optimization. While predicting the future return distributions always represents the most compelling challenge in investment, any underlying distribution can be always well approximated by utilizing a mixture distribution, if we are able to ensure that the component list of a mixture distribution includes all possible distributions corresponding to the scenario analysis of potential market modes. Adopting a mixture distribution enables us to (1) reduce the problem of distribution prediction to a parameter estimation problem in which the mixture weights of a mixture distribution are estimated under a Bayesian learning scheme and the corresponding credible regions of the mixture weights are obtained as well and (2) harmonize information from different channels, such as historical data, market implied information and investors' subjective views. We further formulate a robust mean-CVaR portfolio selection problem to deal with the inherent uncertainty in predicting the future return distributions. By employing the duality theory, we show that the robust portfolio selection problem via learning with a mixture model can be reformulated as a linear program or a second-order cone program, which can be effectively solved in polynomial time. We present the results of simulation analyses and primary empirical tests to illustrate a significance of the proposed approach and demonstrate its pros and cons.

1. Introduction

Pioneered by Markowitz's work on mean–variance portfolio selection (Markowitz, 1952), risk-return analysis frameworks have been now widely adopted in modern portfolio management. While people all agree to take expected returns as one common measure of portfolio performance, there is no consensus on which risk measure best captures investors' risk attitudes, although value-at-risk (VaR, Morgan, 1996; Jorion, 2007) and conditional value-at-risk (CVaR, Pflug, 2000; Rockafellar and Uryasev, 2000, 2002) have become popular risk measures in recent years in portfolio risk management, other than using variances as a risk measure. In practice, however, no matter what type of risk measures is adopted, whether the risk of a portfolio can be well evaluated depends mainly on the reliability and the accuracy of the distribution prediction of asset returns.

* Corresponding author. Tel.: +852 3943 8323; fax: +852 2603 5505.
E-mail addresses: zhuss@mail.sysu.edu.cn (S. Zhu), mjfan@ucdavis.edu (M. Fan), dli@se.cuhk.edu.hk (D. Li).

http://dx.doi.org/10.1016/j.jedc.2014.08.015
0165-1889/

Generally speaking, there are two types of methods for constructing the distribution of asset returns in portfolio selection and risk management, i.e., looking-back methods and looking-forward methods. While the former ones always rely on historical data to fit certain econometric models, the latter ones usually employ financial theories to link the distribution parameters of asset returns in the future with the current market information of these assets (e.g., their market prices). The utilization of looking-back methods is based on a belief that the past behavior pattern of asset returns will be repeated in the future. For example, GARCH model is such a typical approach using historical data to model the dynamic change of the volatility of asset returns (see Bollerslev, 1986). However, this type of methods has sometimes been criticized as driving using the rear-view mirror. The adoption of looking-forward methods is based on a belief that current asset prices are determined, at least partially, by investor expectations on the future. The Black–Litterman approach (Black and Litterman, 1992) for deriving expected asset returns is actually a looking-forward method based on a logic that the market portfolio is the optimal portfolio obtained by mean–variance optimization. Sharpe (2007) also proposed a looking-forward approach using inverse utility maximization to predict the distribution of asset returns, the logic behind which is similar to that of Black and Litterman (1992). Another typical looking-forward method is the approach used in deriving objective distributions from option implied risk neutral distributions based on option pricing theory (see, e.g., Bliss and Panigirtzoglou, 2004; Kostakis et al., 2011).

No matter which kind of methods is adopted in predicting the future, investors always confront with multiple or even conflicting choices of distributions. When using historical data to fit an econometric model, we find that the estimated parameters of the model are often unstable and exhibit different patterns in different time periods. When using option prices to derive implied objective distributions, different choices of pricing kernels lead to different results (see Bliss and Panigirtzoglou, 2004). Furthermore, in practice, the predictions from macroeconomic and industry analyses are often inconsistent, even if not conflicting. Thus, a general paradigm for predicting financial markets in the future always ends up with different scenarios, which also confirms an inherent uncertainty in predicting financial markets in the future due to multiple, albeit plausible, potential choices of distributions.

The question now becomes how to choose a "good" distribution from a candidate list of potential probabilities to predict the future. A mixture model is a natural choice to address such a question. From Wikipedia, we can find an introduction to mixture models as "*In statistics, a mixture model is a probabilistic model for representing the presence of subpopulations within an overall population, without requiring that an observed data-set should identify the sub-population to which an individual observation belongs…. Some ways of implementing mixture models involve steps that attribute postulated sub-population-identities to individual observations (or weights towards such sub-populations), in which case these can be regarded as types of unsupervised learning or clustering procedures*…". Simply speaking, a mixture distribution is a "weighted" distribution that harmonizes different potential distributions (which are usually called mixture components). Furthermore, certain kinds of learning approaches, e.g., Expectation-Maximization (EM) algorithms, can be applied to ascertain the occurrence probability of each component. The readers who are interested in mixture models and the corresponding learning algorithms are referred to McLachlan and Krishnan (1997) and McLachlan and Peel (2001) for more details.

For financial time series, according to the efficient market hypothesis (EMH), only the data that lie within a recent time period can reflect investor anticipations, thus are much more valuable than the data collected a long time ago. However, confining data sources only to recent time periods usually suffers from lack of data. Thus, utilizing learning algorithms, which consider both recent and past data, is a good choice for estimating return distributions. Another reason for choosing mixture models is due to its powerful modeling ability. For example, any distribution can be always well approximated via a mixture of Gaussian distributions (see McLachlan and Peel, 2001). In addition to its simplicity in estimation, Gaussian mixture distributions have become a popular approach to modeling the distribution of asset returns with fat tails, which is a stylized fact in financial data. Mixture models have already been used in modeling the distribution of financial data (see Hall et al., 1989; Peel and McLachlan, 2000).

In investment practices, in addition to the information implied by data, investor views are always necessary to be integrated into the prediction of the future. It was Black and Litterman (1992) who first brought forward such a quantitative approach to adjusting expected returns for qualitative beliefs via Bayesian updating. Note that the Black–Litterman approach only considers integrating investor views into the expected value of returns, without an attempt to adjust the other characteristics of returns, such as the volatility. Bertsimas et al. (2012) revisited the Black–Litterman model from an inverse optimization perspective and developed a more flexible approach to incorporating more investor views, including adjustment of volatilities. Chairawongse et al. (2012) proposed a more flexible approach that can accommodate qualitative views on expected returns expressed in terms of linear inequalities.

One original motivation of Black and Litterman (1992) is to alleviate the fragility of mean–variance portfolio decisions, as they found that the mean–variance portfolio decisions are very sensitive to the means, which indicates that a small error in the estimation of a mean vector might be amplified into a significant change in the corresponding optimal portfolio position. This phenomenon was further authenticated by (Best and Grauer, 1991; Broadie, 1993; Chopra and Ziemba, 1993). In the last decade, recognizing the necessity to consider the robustness of portfolio decisions, many researchers have investigated robust portfolio selection problems under a worst-case analysis framework, resulting in a rich stock of literature. Ben-Tal et al. (1999) proposed a robust multistage asset allocation model to replace the traditional stochastic programming formulation. Costa and Paiva (2002), Goldfarb and Iyengar (2003), Halldórsson and Tütüncü (2003), Garlappi et al. (2007) and Lu (2011) investigated robust mean–variance portfolio selection problems. El Ghaoui et al. (2003), Natarajan et al. (2008) and Zymler et al. (2013) studied robust VaR-based portfolio selection models. Jabbour et al. (2008), Natarajan et al. (2009),