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Adam Smith's invisible hand is unstable: physics and dynamics reasoning applied to economic theorizing

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Abstract

Neo-classical economic theory is based on the postulated, nonempiric notion of utility. Neo-classical economists assume that prices, dynamics, and market equilibria are supposed to be derived from utility. The results are supposed to represent mathematically the stabilizing action of Adam Smith's invisible hand. In deterministic excess demand dynamics, however, a utility *function* generally does not exist mathematically due to nonintegrability. Price as a function of demand does not exist and all equilibria are unstable. Qualitatively, and empirically, the neo-classical prediction of price as a function of demand describes neither consumer nor trader demand. We also discuss five inconsistent definitions of equilibrium used in economics and finance, only one of which is correct, and then explain the fallacy in the economists' notion of 'temporary price equilibria'.

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1. The ideology of unregulated free markets mathematized as rational agents and utility maximization

Standard economics texts begin with the idealization of Adam Smith's invisible hand that is supposed to fulfill agents' wants optimally by matching supply to demand in free, unregulated markets [1,2]. This is implicitly and uncritically an idea of *stability* and equilibrium, e.g., everyone who wants work can find a job, no resources go unused.

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This expectation of equilibrium is maintained by neo-classical economists in spite of the wide disparity with economic data [3]. That the neo-classical program would be optimal for the world is an unjustified assumption, but one that carries enormous political weight today. It has been the main job of neo-classical economists to try to mathematize this idea. The idea of static equilibrium, borrowed from elementary physics, plays the main role in neo-classical economic theorizing.

An equilibrium market can always be defined dynamically correctly [4] as one where the match of supply to demand determines the price of a commodity: the excess demand, defined as demand $D(p)$ minus supply $S(p)$, $\varepsilon(p) = D(p) - S(p)$, vanishes in equilibrium. We point out below that there are other conflicting definitions of equilibrium in the economics and finance literature.

Given n commodities in quantities $x_1,\ldots,x_n$, a function $U(x_1,\ldots,x_n)$, called utility is postulated. Utility is supposed to describe consumer preferences or satisfaction, but only for a so-called 'rational' agent. A rational agent is one who maximizes his own local utility function subject to his budget constraint, which is determined by his total budget M. The function U is usually assumed to be concave, building in the assumption of decreasing returns. This leads to

$$p = \nabla U(x)\,, \tag{1}$$

where price is a function of demand, $p = f(x)$. With decreasing returns this equation leads qualitatively to the following sort of expectations: a man who wants to spend \$100 on a green coat will want to buy one green coat for \$100, two for \$75, three for \$50, four green coats for \$40, and so on. It also predicts that traders place orders for stocks or bonds as follows: A trader will buy 100 shares of cpq at \$10/share, 125 shares at \$8/share, 150 shares at \$6/share, and so on. Now, neither consumers nor traders behave this way. Consumers have demand functions $x = D(p)$ that are step functions: a man will buy a green coat for \$100 or less and while he might possibly be in the market for two green coats, he certainly does not have a demand for 4, 6, 10, or a continuum of green coats. Likewise, demand functions for traders are step functions and these step functions describe the limit buy and sell orders placed in the market [4,5]. In other words, the neo-classical prediction that $p = f(x)$ with decreasing returns does not correctly describe demand in real markets. It also does not describe supply. We turn now to a theoretical criticism of exactly this point, but within the context of neo-classical theory.

There is an old controversy in neo-classical theory: given a price covector $p(x)$ (price as a function of supply or demand x) the integral

$$A_r = \int_C \tilde{p}\,\mathrm{d}x \tag{2}$$

typically depends on the integration path C [6]. Utility is defined by integrating a non-integrable differential form $p\,\mathrm{d}x$, so that utility is mathematically more like 'heat', or a Lagrangian, than it is like potential energy or free energy. Utility is a path-dependent functional, not a function, unless an integrability requirement is satisfied by the differential form. Samuelson [7] tried to argue the integrability problem away but did not solve it. The general problem of nonintegrability has been discussed only in the absence of dynamics and was thought to have been solved by the use of sophisticated