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Discontinuous games with asymmetric information: An extension of Reny's existence theorem ☆

Wei He *, Nicholas C. Yannelis

Department of Economics, Henry B. Tippie College of Business, The University of Iowa, 108 John Pappajohn Business Building, Iowa City, IA 52242-1994, United States

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ABSTRACT

We introduce asymmetric information to games with discontinuous payoffs and prove new equilibrium existence theorems. In particular, the seminal work of Reny (1999) is extended to a Bayesian preferences framework.

1. Introduction

Games with discontinuous payoffs have been used to model a variety of important economic problems; for example, Hotelling location games, Bertrand competition, and various auction models. The seminal work by Reny (1999) proposed the "better reply security" condition and proved the equilibrium existence in quasiconcave compact games with discontinuous payoffs. Since the hypotheses are sufficiently simple and easily verified, the increasing applications of his results has widened significantly in recent years, as evidenced by Jackson and Swinkels (2005) and Monteiro and Page (2008) among others. A number of papers appeared in the topic of discontinuous games and further extensions have been obtained in several directions; see, for example, Lebrun (1996), Bagh and Jofre (2006), Bich (2009), Carbonell-Nicolau (2011), Balder (2011), Carmona (2010, 2011), Carmona (forthcoming), Prokopovych (2011, 2013), Prokopovych (forthcoming), de Castro (2011), McLennan et al. (2011), Reny (2011, 2013), Tian (2012), Barelli and Meneghel (2013) and Prokopovych and Yannelis (2014) among others.

In this paper, we consider discontinuous games with asymmetric information; i.e., games with a finite set of players and each of whom is characterized by his own private information (which is a partition of an exogenously given state space representing the uncertainty of the world), a strategy set, a state dependent (random) utility function and a prior. This problem arises naturally in situations where privately informed agents behave strategically. Because of its importance, the research trend in this field has been quite active since Harsanyi's seminal work. The main purpose of this paper is to provide new equilibrium existence result for Bayesian games with discontinuous payoffs.

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* Corresponding author.
E-mail addresses: he.wei2126@gmail.com (W. He), nicholasyannelis@gmail.com (N.C. Yannelis).

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We introduce the notions of finite/finite* payoff security and adopt the aggregate upper semicontinuity condition in the ex post games. We show that the (ex ante) Bayesian game is payoff secure and reciprocal upper semicontinuous, and hence Reny's theorem is applicable and a pure strategy Bayesian equilibrium exists. A key issue here is that the quasiconcavity of the Bayesian game cannot be guaranteed even if all ex post games are quasiconcave. We show by means of counterexamples that the concavity and finite payoff security conditions of the ex post games are both necessary for the existence of a pure strategy Bayesian equilibrium.[1]

The rest of the paper is organized as follows. In Section 2, we introduce a discontinuous game with asymmetric information and relevant results in deterministic discontinuous games. In Section 3, we prove the existence of a Bayesian equilibrium. Section 4 collects the discussions on the conditions of Theorem 2, the comparison of our notion with the uniform payoff security condition of Monteiro and Page (2007), and possible extensions to the case of a continuum of states. Some concluding remarks are collected in Section 5.

2. Model

2.1. Discontinuous games with asymmetric information

We consider an **asymmetric information game**

$$G = \{\Omega, (u_i, X_i, \mathcal{F}_i)_{i\in I}\}.$$

- There is a finite set of players, $I = \{1, 2, \cdots, N\}$.
- Ω is a countable state space representing the **uncertainty** of the world, $\mathcal{F}$ is the power set of Ω.
- $\mathcal{F}_i$ is a partition of Ω, denoting the **private information** of player i. $\mathcal{F}_i(\omega)$ denotes the element of $\mathcal{F}_i$ including the state ω.
- Player i's action space X_i is a nonempty, compact, convex subset of a topological vector space, $X = \prod_{i\in I} X_i$.
- For every $i \in I$, $u_i : X \times \Omega \to \mathbb{R}$ is a **random utility function** representing the (ex post) preference of player i.

A game G is called a **compact** game if u_i is bounded for every $i \in I$; i.e., $\exists M > 0$, $|u_i(x, \omega)| \le M$ for all $x \in X$ and $\omega \in \Omega$, $1 \le i \le N$. A game G is said to be **quasiconcave (resp. concave)** if $u_i(\cdot, x_{-i}, \omega)$ is quasiconcave (resp. concave) for every $i \in I$, $x_{-i} \in X_{-i}$ and $\omega \in \Omega$. For every $\omega \in \Omega$, the **ex post game** is $G_\omega = (u_i(\cdot, \omega), X_i)_{i\in I}$. Suppose that each player has a **private prior** π_i on $\mathcal{F}$ such that $\pi_i(E) > 0$ for any $E \in \mathcal{F}_i$. The **weighted ex post game** is $G'_\omega = (w_i(\cdot, \omega), X_i)_{i\in I}$, where $w_i(\cdot, \omega)$ is a mapping from X to $\mathbb{R}$ and $w_i(\cdot, \omega) = u_i(\cdot, \omega)\pi_i(\omega)$ for each $\omega \in \Omega$.

For every player i, a **strategy** is an $\mathcal{F}_i$-measurable mapping from Ω to X_i. Let

$$L_i = \{f_i : \Omega \to X_i : f_i \text{ is } \mathcal{F}_i\text{-measurable}\},$$

then L_i is a convex and compact set endowed with the product topology. $L = \prod_{i\in I} L_i$. Given a strategy profile $f \in L$, the **expected utility** of player i is

$$U_i(f) = \sum_{\omega\in\Omega} u_i(f_i(\omega), f_{-i}(\omega), \omega)\pi_i(\omega),$$

then $U_i(\cdot)$ is also bounded by M. Therefore, the **(ex ante) Bayesian game** of G is $G_0 = (U_i, L_i)_{1\le i\le N}$, which is compact and concave if the game G is compact and concave. A strategy profile $f \in L$ is said to be a **Bayesian equilibrium** if for each player i and any $g_i \in L_i$,

$$U_i(f) \ge U_i(g_i, f_{-i}).$$

Remark 1. It is well known that quasiconcavity may not be preserved under summation or integration. Thus, the Bayesian game G_0 may not be quasiconcave even if G is quasiconcave.

2.2. Deterministic case

Hereafter, $G_d = (X_i, u_i)_{i=1}^N$ will denote a **deterministic discontinuous game**, i.e., Ω is a singleton. The following definitions strengthen the notion of payoff security in Reny (1999).

Definition 1. In the game G_d, player i can secure an n-dimensional payoff $(\alpha_1, \cdots, \alpha_n) \in \mathbb{R}^n$ at $(x_i, x^1_{-i}, \cdots, x^n_{-i}) \in X_i \times X^n_{-i}$ if there is $\overline{x_i} \in X_i$, such that $u_i(\overline{x_i}, y^k_{-i}) \ge \alpha_k$ for all y^k_{-i} in some open neighborhood of x^k_{-i}, $1 \le k \le n$.

[1] Based on a different approach using the communication device, Jackson et al. (2002) also studied discontinuous games with asymmetric information.