Journal of Mathematical Economics 46 (2010) 191–213

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Large but finite games with asymmetric information

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ARTICLE INFO

Article history:
Received 1 October 2008
Received in revised form 21 September 2009
Accepted 7 November 2009
Available online 14 November 2009

Keywords:
Nash equilibrium
Approximate equilibria
Large games
Bayesian games
Private information

ABSTRACT

Carmona considered an increasing sequence of finite games in each of which players are characterized by payoff functions that are restricted to vary within a uniformly equicontinuous set and choose their strategies from a common compact metric strategy set. Then Carmona proved that each finite game in an upper tail of such a sequence admits an approximate Nash equilibrium in pure strategies.

Noguchi (2009) and Yannelis (2009) recently proved that a Nash equilibrium in pure strategies exists in a continuum game with asymmetric information in which players are endowed with private information, a prior probability and choose strategies that are compatible with their private information and maximize their interim expected payoffs.

The aim of this paper is to extend Carmona's result to the broader context of Bayesian equilibria and demonstrate that the existence result obtained for continuum games with asymmetric information approximately holds for large but finite games belonging to an upper tail of a sequence of finite games with asymmetric information.

1. Introduction

Schmeidler (1973), in his seminal work on large games, demonstrated that modeling the set of players' names by an atomless measure space obviates the necessity to consider mixed strategies and that a Nash equilibrium in pure strategies exists in a game in which the players are macroscopically negligible and the payoff functions depend on the averages of the strategies chosen by all the players.

Despite a multitude of existence results of Nash equilibria of continuum games, we must keep in mind that continuum is merely an idealized limit of real life situations and as remarked by Mas-Colell and Vives (1993), any result obtained on the basis of such idealization must be tested for robustness near the limit in an approximating sequence.

In this regard, Ali Khan and Sun (1999) proved at first that a continuum game defined on an atomless Loeb space and in which all players face a common compact metric strategy set admits a Nash equilibrium in pure strategies, and proved next that there exists an approximate Nash equilibrium in every finite game in an upper tail of an approximating sequence of finite games, under the condition that the extent to which the distribution of players' characteristics vary is suitably limited.

On the other hand, Carmona (2006) and Carmona and Podczeck (2009) found a way to construct approximate equilibria directly from a Cournot–Nash equilibrium distribution the existence of which is guaranteed under general conditions by Mas-Colell's (1984) existence theorem. To be more specific, Carmona considered an increasing sequence of finite games in which players are characterized by payoff functions that are restricted to vary within a uniformly equicontinuous set and choose their strategies from a common compact metric strategy set. Then Carmona proved that each finite game in an upper tail of such a sequence admits an approximate Nash equilibrium in pure strategies.

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doi:10.1016/j.jmateco.2009.11.004

Carmona's approach is appealing since it obviates the need to deal with measure theoretic complexity pertaining to continuum games of the Schmeidler type and also the need to deal with nonstandard analysis as in Ali Khan and Sun (2002).

Noguchi (2009) and Yannelis (2009) recently proved that a Nash equilibrium in pure strategies exists in a continuum game with asymmetric information in which players are endowed with private information, a prior probability and choose strategies that are compatible with their private information and maximize their interim expected payoffs. As a matter of course, a question arises as to whether Carmona's result can be extended to large but finite games with asymmetric information.

The aim of this paper is to demonstrate that Carmona's approach can indeed be generalized to obtain a similar result for large finite games with asymmetric information after necessary adjustments have been made. We also demonstrate that our version of Carmona's formulation is applicable to real economic situations such as Cournot oligopoly game under uncertainty and environmental economics under uncertainty.

As we justify in the text, we regard the set of the states of the world as a countably infinite set Ω. In order to incorporate interim expected utility maximization into Mas-Colell's original frame work for large anonymous games (Mas-Colell, 1984), it is more convenient to deal with $\mathbb{R}^{\Omega}$-valued payoff functions rather than usual real valued payoff functions. This adjustment requires some modifications on Mas-Colell's original formulation and on the proof of the existence of Cournot–Nash equilibrium distributions.

We also need to investigate in detail properties of our version of "Mas-Colell's transformation", a transformation that is devised to convert an optimization problem with asymmetric information into one with symmetric information.

We remark that there is an undesirable limitation in our model: an approximate Bayesian–Nash equilibrium f is almost optimal for each player t only at the states ω in a *finite subset* $\Omega_0 \subseteq \Omega$. A discussion on this matter can be found in the last section.

The paper is organized in the following manner: Section 1 introduces finite games with asymmetric information. Section 2 presents motivating examples including Cournot oligopoly game under uncertainty and environmental economics under uncertainty along with examples of sequences of finite games which violate the conclusion of our main theorem. Section 3 discusses large non-anonymous games and large anonymous games. Section 4 demonstrates how to associate a game with asymmetric information to a modified large anonymous game and proves the existence of a variant of Mas-Colell's equilibrium distribution. Section 5 states Carmona's result and sketches the proof. Section 6 states and proves our main theorem. Section 7 presents concrete examples to which our results are applicable, and Section 8, the last section, discusses a shortcoming in our results and provides detailed accounts explaining why the shortcoming may be insuperable.

2. Finite games with asymmetric information

Let T be a finite set of players. We assume that all players have an identical *strategy set* A, from which they choose a strategy a. We also assume that all players are uncertain about which state of the world actually obtains but is able to acquire some knowledge about it through their private information. In order to avoid some insurmountable difficulties, we assume as in Yannelis (2009) and Noguchi (2009), that the probability space which represents the uncertainty of the world consists of a countable set of the states of the world $\Omega = \{\omega_1, \omega_2, \omega_3, \ldots\}$ and that the σ-algebra of events in Ω, denoted by $\mathfrak{A}$, equals the power set of Ω, i.e. $\mathfrak{A} = 2^{\Omega}$, on which a probability measure P is defined. For simplicity we assume that there are no P-null events in $\mathfrak{A}$ except the empty event $\varnothing$.

Private information of a player is simply a sub σ-algebra $\mathfrak{F} \subseteq \mathfrak{A}$. Let $\mathcal{X}$ be a strategy*space*, which consists of all strategies. For technical reasons, we assume that the strategy space $\mathcal{X}$ is a *separable Banach space* and A a *non-empty*, *weakly compact*, and *convex* subset of $\mathcal{X}$. Let $L(\Omega, \mathcal{X})$ be the Banach space of $\mathcal{X}$-valued Bochner integrable functions on Ω and define

$$L(\Omega, A) = \{x \in L(\Omega, \mathcal{X}) : x(\omega) \in A,\ \forall \omega \in \Omega\}.$$

Note that by Diestel' theorem (Yannelis, 1991, Theorem 3.1, p. 7), $L(\Omega, A)$ under the present assumptions is weakly compact, convex, and metrizable (Dunford and Schwartz, 1958, Theorem V.6.3, p. 434). In the sequel we fix a metric that is compatible with the topology of weak convergence of probability measures on $L(\Omega, A)$ and denote this metric by ρ.

For each private information $\mathfrak{F}$, define the subset $L^{\mathfrak{F}}(\Omega, A) \subseteq L(\Omega, A)$ consisting of all $\mathcal{X}$-valued Bochner integrable functions that are $\mathfrak{F}$-measurable. In the sequel, $E(\cdot|\mathfrak{F})$ denotes the conditional expectation operator with respect to $\mathfrak{F}$, which is known under the present assumptions to exist for $L(\Omega, \mathcal{X})$ and to be a linear contraction projection from $L(\Omega, A)$ to $L^{\mathfrak{F}}(\Omega, A)$ (Diestel and Uhl, 1977, Corollary 8, p. 48; Theorem 4, p. 123).

Remark 1. Note in light of Diestel and Uhl (1977, Corollary 8, p. 48; Theorem 4, p. 123) that it is indispensable to impose the condition that A is convex in order to guarantee that $E(\cdot|\mathfrak{F})$ respects $L(\Omega, A)$ i.e., $E(\cdot|\mathcal{F})[L(\Omega, A)] \subseteq L(\Omega, A)$. Also note that because of this convexity condition on A our main result does not entirely subsume the complete information result of Carmona and Podczeck (2009).

An element $x \in L(\Omega, A)$ is said to be a *random strategy* and an element $x \in L^{\mathfrak{F}}(\Omega, A)$ a *random s* trategy compatible with $\mathfrak{F}$. A function $g : T \to A$ is said to be a *strategy profile* and a function $f : T \to L(\Omega, A)$ a *random strategy profile*. Note that if ξ_ω denotes the evaluation map at $\omega \in \Omega$, the composition of ξ_ω with a random strategy profile f gives rise to a strategy profile $\xi_\omega \circ f$. We remark that by the assumptions on $(\Omega, \mathfrak{A}, P)$, the evaluation map $\xi_\omega : L(\Omega, \mathcal{X}) \to \mathcal{X}$ is well-defined and bounded linear since $\|x(\omega)\|_{\mathcal{X}} \leq \|x\|_{L(\Omega,\mathcal{X})} P(\{\omega\})^{-1}$ for all $x \in L(\Omega, \mathcal{X})$. In particular, ξ_ω is weakly continuous.