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An economic order quantity model for an imperfect production process with entropy cost

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ABSTRACT

Among the assumptions of the classical economic order quantity (EOQ) model is that all units that are purchased (or produced) are of perfect quality. However, this is frequently unrealistic since production processes deteriorate resulting in the production of defective products requiring rework. Some recent studies suggest that production systems performance might be improved by applying the first and second laws of thermodynamics to reduce system entropy (or disorder). This paper applies the concept of entropy cost to extend the classical EOQ model under the assumptions of perfect and imperfect quality. Mathematical models are developed and numerical examples illustrating the solution procedure are provided. Accounting for entropy cost suggests that order quantities should be larger than the figures derived from the classical EOQ model.

1. Introduction

The economic order quantity (EOQ) model developed by Harris (1915) is believed to be the earliest published inventory management model and has been the cornerstone for the development of many inventory models since then. The assumptions of the EOQ model make its mathematics easy to understand and use. However, even though the EOQ model has been widely used as a decision-making tool in practice, the assumptions necessary to justify use of the model often were not met (Osteryoung et al., 1986; Woolsey, 1990). For example, among the assumptions of the EOQ model is that all units that are purchased or produced are of perfect quality. This may be an unrealistic assumption since the product quality is directly affected by the reliability of the production process (Cheng, 1991).

In practice, most manufacturing processes are not defect free and result in items that require rework. Gopalan and Kannan (1995, p. 935) wrote: "All over the world, industries are concentrating in making quality an inherent in their products. In spite of these efforts, rework is becoming an unavoidable factor in many production systems. For example, glass manufacturing, food processing, etc." Agnihothri and Kenett (1995, p. 308) wrote: "Although 'doing it right the first time' is an important goal to pursue, defects and reworks are common occurrences in a manufacturing process." They provided an example encountered on the production floor of a printed circuit board manufacturing industry. Geren and Redford (1999, p. 159) wrote: "Printed circuit board assembly (PCBA) rework is an acceptable process step in PCBA manufacturing and widely performed using manual and semi-automated tools." Thus, the possibility of the process going out-of-control is valid, and efforts to rework defective items must be accounted for.

Among the earliest models that link the EOQ model and product quality are those of Porteus (1986) and Rosenblatt and Lee (1986). Both of these papers assume

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that the process is in control at the beginning of production and may go out of control during production, with a constant probability, resulting in defective items requiring rework. Rosenblatt and Lee (1986) assume that the time for the process to go out-of control is an exponential random variable. Porteus (1986) and Rosenblatt and Lee (1986) assume that once the process is out-of-control, it remains in that state until the complete lot has been produced, and that the defective units generated are not discarded but reworked. Their results suggest producing in smaller lots to attain lower defect levels. Other researchers found the opposite results. Garvin (1988) and Bourland et al. (1997) found that the percentage of acceptable parts increases with larger lots (Urban, 1998). In practice, one is usually interested in manufacturing in smaller batches, which although increasing the control costs (entropy cost), provide greater flexibility.

Jaber et al. (2004) applied classical thermodynamics reasoning to model management systems, and applied the concept of entropy cost when modelling inventory systems. Separately, the thermodynamic entropy concept has been applied to analysing decision trees (Drechsler, 1968), manpower systems (Tyler, 1989), logistics management (Whewell, 1997), business process management (Chen, 1999), the price–quality relationship (Nuwayhid et al., 2006), coordinating orders in a supply chain (Jaber et al., 2006) and reverse logistics (Jaber and Rosen, 2008).

Jaber et al. (2004) postulated that the behaviour of production systems closely resembles that of physical systems. They suggested that production system performance could be improved by applying the first and second laws of thermodynamics to reduce system entropy. They also suggested using the concept of entropy cost to account for the hidden costs inherent in management systems. Examples of these costs found in the literature include managerial cost to control the improvement process (Ullmann, 1982), the difficulties in maintaining a steady stream of incoming materials (Crusoe et al., 1999), and product return costs and obsolescence costs (Callioni et al., 2005). Jaber et al. (2004) used the economic order (production) quantity (EOQ/EPQ) model to illustrate the applicability of the first and second laws of thermodynamics to production systems. The numerical results showed that the cost to control the commodity outflow (entropy cost) is higher for the classical than it is for the suggested model and, provided that including an entropy cost is appropriate, this observation suggests that large lots are cheaper to control than small ones.

This paper investigates production processes that generate defects requiring rework. It does this by using an EOQ model with entropy costs. The paper adopts the approach of Porteus (1986) to estimate the number of defectives per lot.

The remainder of this paper is organised as follows. Section 2 provides a brief introduction to the laws of thermodynamics, the concept of entropy cost and commodity flow strategies. Section 3 uses mathematical modelling to extend the classical ECOQ model by incorporating the entropy cost associated with the commodity flow strategy suggested in Section 2. Section 3 also provides a brief description of the mathematics for the EOQ model with rework developed by Porteus (1986), and extends this model by accounting for entropy cost. Section 4 provides numerical examples and discusses the results. The paper concludes in Section 5.

2. Thermodynamics and the concept of entropy

Jaber et al. (2004) postulated that a production system resembles a physical system operating within surroundings, which include the market and the supply system. A physical thermodynamic system is defined by its temperature, volume, pressure and chemical composition. A system is in equilibrium when these variables have the same values at all points. A production system could be described analogously by its characteristics, for example the price (P) that the system ascribes to the commodity (or collection of commodities) that it produces.

Reducing the price of the commodity below the market price may increase demand and produce a commodity flow (sales) from the system to its surroundings. This is similar to the flow of heat from a high temperature (source) to a low temperature (sink) in a thermodynamic system, where part of this heat is converted into useful work and some of the heat is lost from the system and wasted. Analogously, as Fig. 1 shows, some of the commodity is sold to the market and is converted to revenue (equivalent to useful work) and some is wasted (hidden cost).

To guarantee a commodity flow (demand) from the inventory system to the market, the following two strategies are considered. The first strategy suggests that a firm may provide the same quality product as its competitors but at a lower price; the second strategy suggests that a firm provides a better quality product than its competitors at the same price. These two strategies agree with Whewell (1997, p. 18), who suggested that a supplier that seeks to improve its share of the market, will reduce costs associated with providing the same level of satisfaction and/or attempt to increase or sustain customer satisfaction relative to competitors. Furthermore, Whewell (1997) advocated that changes to improve customer satisfaction can alter established practices in at least one part of the supply process. Such changes can create disorder (entropy), the costs of which are difficult to quantify. Like Jaber et al. (2004), this paper adopts the first strategy where the suggested commodity flux, or demand rate, is of the form

$$\delta(t) = -K(P(t) - P_0(t)) \tag{1}$$

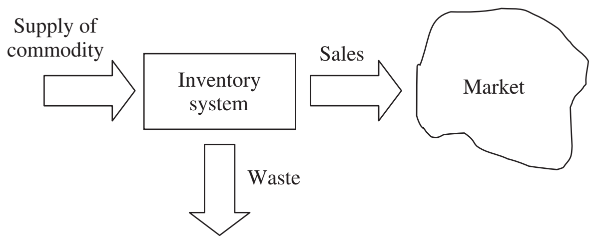

Fig. 1. The inventory system and its surrounding.