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Economic order quantity model for items with imperfect quality with learning in inspection

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ABSTRACT

The economic order quantity (EOQ) model is the simplest and earliest inventory model in the literature. Its simple mathematics is attributed to its assumptions, which are rarely met. Salameh and Jaber [2000. Economic production quantity model for items with imperfect quality. Int. J. Prod. Econ. 64, 59–64] addressed one of these assumptions that items received or produced are not of perfect quality. This paper extends Salameh and Jaber's work for the case where there is learning in inspection. The model is realistic in that it considers situations of lost sales and backorders. Mathematical models are developed with numerical examples provided and results discussed for the cases of (i) partial transfer of learning, (ii) total transfer of learning, and (iii) no transfer of learning.

1. Introduction

The EOQ is the earliest, simplest and most appreciated inventory model in the literature (Osteryoung et al., 1986). Its popularity amongst academicians and businesspeople has been attributed to the ease of manipulation and calculation (Woolsey, 1990). However, some of its assumptions are never met in practice (e.g., Jaber et al., 2004). One of these assumptions is that items ordered (or produced) are of perfect quality (Cheng, 1991).

There are several studies in the literature that investigated the affect of imperfect quality on the inventory policy. Porteus (1986) and Rosenblatt and Lee (1986) are two of the earliest of these studies. These studies have been the cornerstone for many models, which assumed that defective items are reworked. The defective items cause a manufacturer a rework cost. They also result in the shortage of some of the items. These shortages can be treated as lost sales or backorders by a vendor. Montgomery et al. (1973) studied a periodic review inventory model where lost sales and backorders are caused by the stock-out of inventory in a production system. Nahmias and Smith (1994) discussed a similar model with partial lost sales. They assumed instantaneous deliveries from the warehouse to the retailers. Ouyang et al. (1996) extended a model in the literature for the case where shortages are allowed. They studied a continuous review inventory model where both lead time and order quantity are taken as decision variables. Chiu (2003) studied a manufacturing setting that produces defective items that are either scrap or repairable. He allowed for backorders and formulated expressions for the optimal production and backorder sizes. Liu et al. (2007) discussed a lot sizing problem with limited inventory capacity, which is smaller than production capacity. They did not allow for backlogging while the unmet demand was taken as lost sale. Hill et al. (2007) considered lost sales in a single item two-echelon supply chain where the retailers demand follows Poisson distribution while the warehouse lead time is fixed.

Along the line of research of Porteus (1986) and Rosenblatt and Lee (1986) but with a different treatment of imperfect quality items, Salameh and Jaber (2000) assumed that these items are withdrawn and sold as a single batch by the end of the screening process. The work of Salameh and Jaber (2000) has been receiving increasing attention by researchers in the field. This interest is reflected in the number of citations it has received (e.g., *Scopus* and *Scholar Google*). Following is a brief survey.

Cárdenas-Barrón (2000) observed and corrected a minor error that does not affect the main idea and the remainder of the article. Goyal and Cárdenas-Barrón (2002) presented a simpler approach for determining the economic production quantity for an item with imperfect quality. Rezaei (2005) brought in the concept of shortages and compared the expected profit with that of an EOQ model. Papachristos and Konstantaras (2006) clarified a point related to the condition for preventing stock-outs and extended the model to the case in which withdrawing takes place at the end of the planning horizon. Eroglu and Ozdemir (2007) and Wee et al. (2007) independently extended the work of Salameh and Jaber (2000) to account for backorders. When dealing with imperfect quality items, Konstantaras et al. (2007) examined two options: either to sell them as a single batch at a discounted price or rework them at a cost. Maddah and Jaber (2008) rectified a flaw in Salameh and Jaber's work related to the method of evaluating the expected profit per unit time. Jaber et al. (2008) assumed the

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percentage defective per lot reduces according to a learning curve, which was empirically validated by the data from automotive industry. Recently, Maddah et al. (2009) considered a production/inventory system where items produced/purchased are of two different qualities. Hsu and Yu (2009) investigated the work of Salameh and Jaber (2000) under a one-time-only discount. These works are either direct extensions or modifications to the work of Salameh and Jaber (2000). Others have extended the work of Salameh and Jaber (2000) either by investigating it in a supply chain context (e.g., Huang, 2004; Chung and Huang, 2006, Chung et al., 2009) or by applying fuzzy set theory (e.g., Chang, 2004; Wang et al., 2007; Björk, 2009).

One fertile area in this line of research is that the production facilities do overcome their shortages through a human phenomenon known as learning. That is, the workers tend to produce faster as they spend more time on the same machine in a line. This natural phenomenon was shown by Wright's (1936) learning curve. With an interruption in the production, the workers tend to forget part of their skills. The earliest work in the literature that investigated the lot sizing problem with learning and forgetting is that of Keachie and Fontana (1966). Since there has been some interest in this subject, Jaber and Bonney (1999) provided almost a comprehensive survey (for the period 1966–1998). Some of the works in this line of research are, but not limited to, Balkhi (2003), Chiu et al. (2003), Chiu and Chen (2005), Jaber and Guiffrida (2007), Alamri and Balkhi (2007), and Jaber and Bonney (2003, 2007, 2009). Although there is consensus among these works on the form of the learning curve, it was not so for forgetting (Jaber, 2006). However, Jaber and Bonney (1996) developed a learn-forget curve model (LFCM) that properly represents the learning-forgetting process (Jaber and Bonney, 1997; Jaber et al., 2003; Jaber and Sikström, 2004).

There are situations where the time to inspect defective items follows a learning curve (e.g., Sikström and Jaber, 2002). None of the above surveyed articles and those available in the literature investigated the model of Salameh and Jaber (2000) for learning in inspection. This paper addresses this research gap and extends Salameh and Jaber's model in two directions. First, this paper considers that the screening rate of defectives follows a learning curve where stock-out may occur when the screening rate is slower than the demand rate. These stock-outs can be treated as either lost-sales or backorders. Second, this paper includes the transfer of knowledge in learning when the production moves from one cycle to another in three possible scenarios: (i) no transfer of learning, (ii) total transfer of learning, and (iii) partial transfer of learning.

The remainder of this paper is organized as follows. Section 2, provides a brief introduction to the model of Salameh and Jaber (2000). Section 3 provides a background to the learning and forgetting theory. Section 4 is for mathematical modelling. Section 5 provides numerical examples and discusses results. Finally, this paper concludes in Section 6.

2. The model of Salameh and Jaber (2000)

Salameh and Jaber (2000) extended the traditional EOQ model by accounting for imperfect quality items. They assumed a 100% screening and poor-quality items are withdrawn from inventory by the end of the screening period, τ, and are sold as a single batch at a discounted price. The behavior of inventory is as described in Fig. 1, where y is the lot size quantity, p is the fraction of defectives and $f(p)$ is its probability density function, while T is the cycle time.

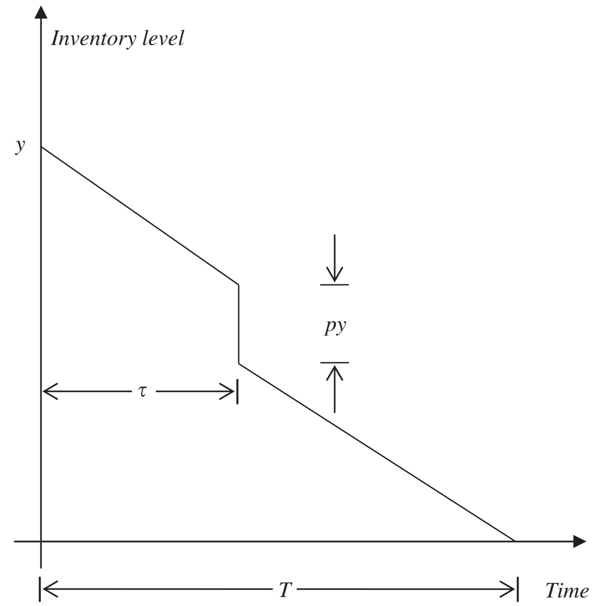

Fig. 1. Behavior of the inventory with time for the model of Salameh and Jaber (2000).

The expected per unit time profit function was given as

$$E[TPU](y) = D(s - v + hy/x) + D(v - hy/x - c - d - K/y)E\left[\frac{1}{1-p}\right] - \frac{hy(1-E[p])}{2} \tag{1}$$

where $p \sim [a, \ b]$, $E[p] = \int_a^b pf(p)\,dp$, $E[1/(1-p)) = \int_a^b (1/(1-p))f(p)\,dp$, D is the demand rate, s is the unit selling price of items of good quality, c is the unit purchase cost, v is the unit selling price of defective items ($v < c$), x is the screening rate ($x > D$), d is the unit screening cost, K is the order cost and h is the unit holding cost. To guarantee there are no shortages, Salameh and Jaber (2000) set the condition $p \le D/x$. The optimal *order* quantity, y^*, that maximizes (1) was given as (Cárdenas-Barrón, 2000)

$$y^* = \sqrt{\frac{2KDE[1/(1-p)]}{h[1 - E[p] - 2D(1 - E[1/(1-p)])/x]}} \tag{2}$$

3. Learning and forgetting

Learning is a natural phenomenon that occurs as a worker performs a task repetitively. The data from a learning curve fits well to the power form of learning suggested by Wright (1936). That is

$$\pi_n = \pi_1 n^{-b} \tag{3}$$

where π_1 is the time to perform the first repetition b is the learning exponent $0 < b < 1$, and n is the cumulative number of repetitions. For review of learning curves, reader may refer to Jaber (2006).

Forgetting curve (Jaber and Bonney, 1996) is usually taken to be a mirror image of the learning curve. To determine the forgetting exponent f_i in cycle i, it is customary to equate the learning and forgetting time at the instant a worker has inspected y_i units. This determines the value of the intercept of the forgetting curve $\overline{\pi_1}$. The forgetting curve takes the form

$$\overline{\pi}_m = \overline{\pi}_1 m^f \tag{4}$$