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Economic ordering quantity models for items with imperfect quality

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Abstract

In this paper, we first look at the issue of non-shortages in models with proportional imperfect quality, when the proportion of the imperfects is a random variable. More specifically we revisit the papers of Salameh and Jaber (Int. J. Prod. Econom. 64 (2000) 59) and Chan et al. (Prod. Plann. Control 14 (2003) 588) and we point out that the sufficient conditions given in these papers to ensure that shortages will not occur may not really prevent their occurrence. Next, considering the timing of withdrawing the imperfect quality items from stock, we clarify a point not clearly stated in Salameh and Jaber (Int. J. Prod. Econom. 64 (2000) 59) and we extented this model to the case in which withdrawing takes place at the end of the planning horizon. Finally, we establish that the objective of maximizing the mean average profit, which was set in Salameh and Jaber (Int. J. Prod. Econom. 64 (2000) 59), is superfluous and the results given there can be obtained by ignoring the revenue issues assumed in the model and minimizing directly the mean average cost.
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1. Background

It is generally admitted that inventory management is affected by the imperfect quality of goods and this aspect has to be taken into account. The issue of what quality is and how quality is defined and managed is a big one and we are not going to discuss it here. A good account on this topic is given in the book by Nahmias (2001), where an extensive list of references related to the topic is also given.

How can we model the imperfect quality? One approach, the simplest and most frequently used, is to suppose that a certain percent, say p of goods coming into the warehouse are defective. The following scenarios are possible for the parameter p. It may have a specific value, in which case the system is deterministic. It may be a random

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variable, in which case the system is stochastic, having a completely known distribution, a known distribution dependent on an unknown parameter or an unknown distribution. This modeling for defectives is known in the literature as "proportional imperfect quality". A good account as to how this proportional imperfect quality modeling affects inventory systems can be found in the book by Zipkin (2000).

2. Introduction

Models with proportional imperfect quality have attracted the interest of researchers and there are many papers dealing with this topic. However, since the main objective of this paper is to discuss the issue of non-shortages in a class of such models, we skip from going into a literature review and come to the point. Salameh and Jaber (2000) studied an EOQ model having the following characteristics. Each EOQ lot contains a certain percentage of defective items. This percentage is a continuous random variable with completely known probability density function. As soon as the lot comes into the warehouse, a 100% percent inspection process of the lot is conducted, at a fixed cost per unit and at a rate greater than that of the demand. Demand occurs parallel to the inspection process and is fulfilled from goods found to be perfect by the inspection process. The model does not allow for shortages and a sufficient condition is given to ensure it. Units found to be imperfect, items of poor quality as they are usually called, are kept in stock and sold at a price lower to that of perfect quality units. To this end the authors assume the following:

(a) "Items of poor quality are kept in stock and sold prior to receiving the next shipment as a single batch at a discounted price of u per unit" (p. 61).
(b) "The behavior of the on hand inventory level is illustrated by Fig. 1" (p. 61), which is exactly Fig. 1 of this paper.

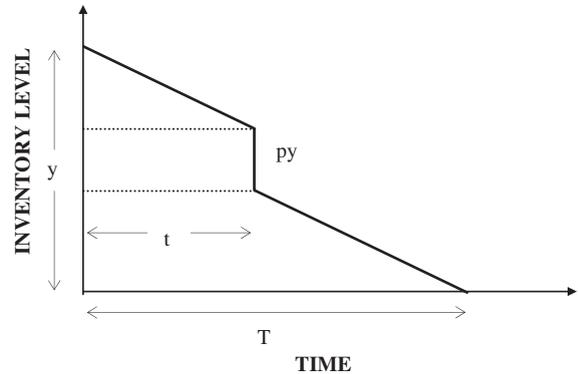Chan et al. (2003) also studied a model, with a structure quite similar to that in Salameh and

Fig. 1. Inventory level fluctuation as given in Salemeh and Jaber's paper.

Jaber (2000), and with non-shortages. The authors also give a condition, which is declared as sufficient to prevent shortages. These are the main assumptions on the above papers, while for the minor ones we refer to the papers themselves.

The notation used by Salameh and Jaber (2000), which will also be adopted for Chan et al.'s (2003) model is the following:

D	demand per unit of time
y	order size
c	unit variable cost
K	fixed cost of placing an order
p	random variable representing the percentage rate of defective items received
$f(p)$	probability density function of p
s	unit selling price of items of good quality
u	unit selling price of defective items, $u < c$
x	inspection rate
d	unit inspection cost
t	the inspection time
T	cycle length
M	expected value of $1/(1 - p)$
ETPU(y)	total profit per unit of time when the cycle's lot size is y

Working on their model Salameh and Jaber (2000) obtained the optimal lot size by maximizing ETPU(y). Related to this work is the paper by