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A comprehensive extension of an integrated inventory model with ordering cost reduction and permissible delay in payments

Kuo-Ren Lou *, Wan-Chih Wang

Department of Management Sciences, Tamkang University, Tamshui Dist., New Taipei City, Taiwan, ROC

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ABSTRACT

Huang (2010) [1] proposed an integrated inventory model with trade credit financing in which the vendor decides its production lot size while the buyer determines its expenditure to minimize the annual integrated total cost for both the vendor and the buyer. In this paper, we extend his integrated supply chain model to reflect the following four facts: (1) generated sales revenue is deposited in an interest-bearing account for the buyer, (2) the buyer's interest earned is not always less than or equal to its interest charged, (3) the total number of shipments in one lot size is the vendor's decision variable to minimize the cost, and (4) it is vital to have a discrimination term which can determine whether the buyer's replenishment cycle time is less than the permissible delay period or not. We then derive the necessary and sufficient conditions to obtain the optimal solution, and establish some theoretical results to characterize the optimal solution. Finally, numerical examples are presented to illustrate the proposed model and its optimal solution.

1. Introduction

In practice, vendors usually offer their buyers a permissible delay in payments. During the permissible delay period, there is no interest charge. Hence, buyers can earn the interest from sales revenue meanwhile vendors lose the interest earned during the same period. However, if the payment is not paid in full by the end of the permissible delay period, then vendors charge buyers an interest on the unpaid amount of the purchasing cost. The permissible delay in payments produces two benefits to the vendor: (1) it attracts new buyers who consider it to be a type of inventory cost reduction, and (2) it is an alternative to price competition because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions. On the other hand, the policy of granting trade credit financing adds not only an additional opportunity cost but also an additional dimension of default risk to the vendor.

Goyal [2] developed an economic order quantity (EOQ) model for the buyer when the seller offers a permissible delay in payments. Shah [3] considered a stochastic inventory model when delays in payments are permissible. Jamal et al. [4] extended Goyal's model to allow for shortages. Hwang and Shinn [5] added the pricing strategy to the model, and derived the optimal price and lot sizing for a retailer under the condition of permissible delay in payments. Teng [6] provided an alternative conclusion from Goyal [2], and proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Chang et al. [7] developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Huang [8] extended Goyal's model to develop an EOQ model in which the supplier offers the retailer the up-stream trade credit period M, and the retailer in turn provides the down-stream trade credit period N (with $N \leqslant M$) to his/her customers. Teng and Goyal [9] complemented the shortcoming

* Corresponding author. Fax: +886 2 8631 3214.
E-mail address: 109880@mail.tku.edu.tw (K.-R. Lou).

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of Huang's model and proposed a generalized formulation. Teng [10] established an EOQ model for a retailer who offers distinct trade credits to its good and bad credit customers. Chang et al. [11] presented the optimal manufacturer's replenishment policies in a supply chain with up-stream and down-stream trade credits. Hu and Liu [12] established an optimal replenishment policy for the economic production quantity (EPQ) model with permissible delay and allowable shortages. Teng et al. [13] obtained the retailer's optimal ordering policy when the supplier offers a progressive permissible delay in payments. Recently, Yang and Chang [14] proposed a two-warehouse partial backlogging inventory model for deteriorating items with permissible delay under inflation. Many related articles can be found in [15,11,16–27,13], and their references.

Recently, Huang [1] developed an integrated inventory model to determine the optimal policy with ordering cost reduction and permissible delay in payments. In this paper, we extend his integrated supply chain model to reflect the following four facts. (1) The buyer deposits sales revenue into an interest-bearing account which is stated in his Assumption 4. Thus, the buyer's interest earned is based on sales revenue, not based on purchasing cost as shown in Huang [1]. (2) The interest earned is not always less than the interest charged. For instance, one wants to borrow money from a bank simply because one thinks the interest earned could be higher than the interest charged. Hence, we relax his dispensable assumption that the buyer's interest earned is always less than or equal to its interest charged, (3) To minimize the cost, the vendor should treat the total number of shipments in one lot size as a decision variable instead of a constant as in Huang [1]. (4) It is helpful to have a discrimination term which can determine whether the buyer's replenishment cycle time is less than the permissible delay period or not. We then derive the necessary and sufficient conditions to obtain the optimal solution, and establish some theoretical results to characterize the optimal solution. Finally, numerical examples are presented to illustrate the proposed model and its optimal solution.

2. Mathematical formulation

For simplicity, we use the same assumptions and notation as those in Huang [1]. However, for generality, we let s be the unit selling price, and treat n as a decision variable. In addition, we also relax the following two dispensable assumptions of $I_k \geqslant I_e$, and $M \geqslant N$ as in Huang [1].

The annual integrated total cost consists of the vendor's annual total cost and the buyer's total cost. We discuss them separately below.

2.1. The vendor's annual total cost

The total number of shipments in one production lot size (i.e., n) is the only decision variable for the vendor. Hence, the vendor's annual total cost is a function of n. From Fig. 1, we know that the vendor's on-hand inventory is equal to the area of the shaded region OEFGBD.

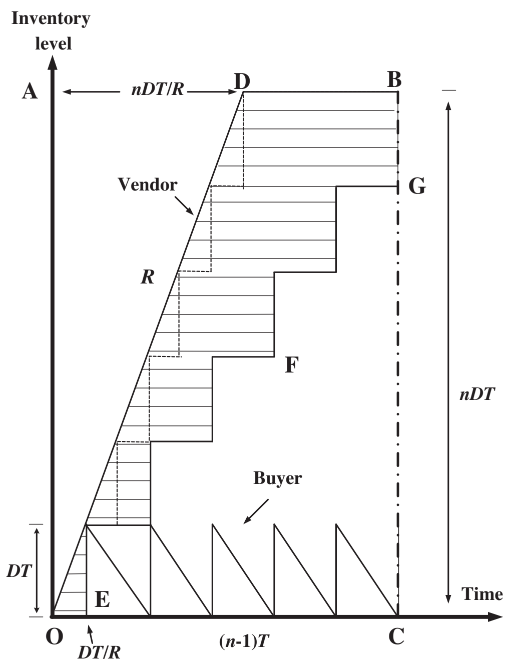

Fig. 1. The vendor's on-hand inventory.