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Maximizing profits in an inventory model with both demand rate and holding cost per unit time dependent on the stock level ☆

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ABSTRACT

We study an EOQ inventory model with demand rate and holding cost rate per unit time, both potentially dependent on the stock level. The ordering cost, the holding cost and the gross profit from the sale of the item are considered. The objective is to maximize the average profit per unit time. We present the analytical formulation of the problem and demonstrate the existence and uniqueness of the optimal cycle time, giving a numerical algorithm to obtain it. Moreover, we provide two fundamental theoretical results: a rule to check when a given cycle time is the optimal policy, and a necessary and sufficient condition for the profitability of the system. Several EOQ models analyzed by other authors are particular cases of the one here studied. We present some numerical examples to illustrate the proposed algorithm and analyze the sensitivity of the optimal solution with respect to changes in various parameters of the system.

1. Introduction

Historically, the first mathematical models to study inventory systems consider that the holding cost rate per item and per unit time is constant. Thus the holding cost is proportional to the value obtained from multiplying the number of available items by the time period they are in stock. This allows a simple formulation of the problem. Later on, practitioners note that sometimes the holding cost per item increases with the time in a non-linear way and different models consider this option in their formulation. For example, Naddor (1982, p. 106), and Weiss (1982) are classical works which follow this approach. They suppose that the holding cost per item is a convex potential function of time. Also, they argue that this case can occur for perishable goods such as foodstuffs, milk, fruit, vegetables and meat, whose quality drops with each passing day and, as a result, increasing holding costs are necessary to maintain the freshness of the items and to prevent spoilage. Ferguson, Jayaraman, and Souza (2007) provide a real-life application of the model given by Weiss (1982) and propose a simple methodology to estimate the holding cost curve parameters using data from real stores. In the literature we can find other EOQ models with time-dependent holding costs (see, for instance, Fujiwara and Perera (1993), Alfares (2007) and Roy (2008)).

Similarly, some inventory managers assume that the holding cost per unit time is linearly proportional to the amount held in stock. However, this holding cost per unit time may increase (or decrease) as quantities increase. This situation can occur when the value of the inventory item is very high and many precautionary steps have to be taken to ensure its safety and quality; for example, in carrying luxury items like expensive jewellery and designer watches, electronic components, radioactive substances or volatile liquids. In this sense, Naddor (1982, p. 107) introduces the idea of non-linear holding costs in quantity and develops a model with constant demand rate for this situation, which is known as "the expensive-storage system". Later, Goh (1994) considers the same situation with stock-dependent demand rate for the problem of minimizing the total inventory cost per unit time in a model that he titles "instantaneous replenishment with non-linear stock dependent carrying cost", where the holding cost rate per unit time is a convex potential function of the items in stock. Giri and Chaudhuri (1998) extend the previous model to cover an inventory for a deteriorating item where the rate of deterioration is a small constant fraction of the inventory level. Recently, Mahata and Goswami (2009) generalize the model of Giri and Chaudhuri (1998) to the case of fuzzy deterioration rate. Except for the cited papers, as far as the authors know, EOQ models with non-linear stock-dependent holding costs are scarce.

Another issue noted by marketing researchers and practitioners is that an increase in inventories may bring about increased sales

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of some items. Thus, many mathematical models for inventory systems consider the use of stock-dependent demand rate in their formulation. Baker and Urban (1988) develop this idea, assuming that the demand rate is a concave potential function of the inventory level and raising the issue from the perspective of maximizing the profit per unit time. Since then, many papers have appeared considering the demand rate as a function of the stock level, but most of them approach the problem from the perspective of minimizing the total inventory cost per unit time. For example, the cited papers of Goh (1994), Giri and Chaudhuri (1998) and Mahata and Goswami (2009) follow this goal. We refer the reader to Urban (2005) for a more detailed review of the related literature.

In the inventory models, the objective function to be minimized is usually the total cost related to the inventory. However, in real life, the main goal of inventory management is to maximize profit and, consequently, we need to include revenues and purchasing costs in the objective. For this reason, some authors have recently considered inventory models with maximizing of profits per unit time. Padmanabhan and Vrat (1995) and Chung, Chu, and Lan (2000) analyze models with profit maximizing for perishable and deteriorating items. Jung and Klein (2006) use geometric programming to obtain the optimal solutions for three inventory models under profit maximization. Sana and Chaudhuri (2008) study models of maximum profit considering delays in payments and price-discount offers with different possibilities for the demand rate. Urban (2008) analyzes an extension of the inventory models with discretely variable holding costs and considering profit maximization. Other works focusing on the maximizing of profits are Dye and Ouyang (2005), Teng and Chang (2005), San-José, Sicilia, and García-Laguna (2007) and Roy (2008).

In this paper, we formulate an inventory model in which: (i) the holding cost per unit time is a convex potential function of the stock level; (ii) the demand rate is a concave potential function of the inventory level and (iii) the objective consists of maximizing the profit per unit time. We present an algorithm to obtain the optimal cycle time and the optimal profit. We also establish a rule to characterize this optimal solution and provide a necessary and sufficient condition to determine whether the inventory is profitable, using the initial parameters of the model. Several inventory models studied by other authors are particular cases of the model analyzed here.

The structure of this paper is as follows. In Section 2, we introduce the formulation of the model, the assumptions, the notation to be used throughout the article and the objective function to be maximized. Section 3 deals with the model optimization and presents the algorithm for determining the optimal inventory policy. In Section 4, we include some interesting properties, various particular models which can be obtained using our inventory system and a comparison of the optimal solution with the solution of minimum total inventory cost. Numerical examples and sensitivity analysis are shown in Section 5. Finally, the conclusions are set out in Section 6.

2. Model formulation

2.1. Notation

We use the following notation throughout the paper:

T	length of the inventory cycle (>0, the decision variable);
q	order quantity or lot size per cycle (>0);
$I(t)$	level of inventory at time t ($\leqslant q$);
$D(t)$	demand rate function;
λ	scaling constant for demand rate (>0);
β	inventory level elasticity of demand rate ($0 \leqslant \beta < 1$);
K	ordering cost per order (>0);
p	unit purchasing cost (>0);
s	unit selling price ($\geqslant p$);
$H(x)$	holding cost per unit time and per x stored items;
h	scaling constant for holding cost (>0);
γ	holding cost elasticity ($\geqslant 1$).

Note. We assume the more general condition $s \geqslant p$, instead of $s > p$, because this assumption allows several inventory models presented by other authors to be included in our study; that is, the inventory models that were developed under the perspective of minimizing the sum of the ordering cost and average holding cost. These models can be called *models of minimum cost*. Formally, they can be obtained from our inventory model by taking $s = p$. However, as our goal is to find the policy that maximizes the profit per unit time, we need to include the revenues and purchasing costs in the model. So, we call this class *models of maximum profit*. Of course, a *model of minimum cost* (which does not consider revenues and purchasing costs) studies only a subsystem of the inventory system associated to the *model of maximum profit.* Although in a few models, such as the classical EOQ model, solving the first type of systems is equivalent to solving the second though, in general, this is not true. For example, if we solve the *model of minimum cost* developed by Goh (1994) and, later on, we add revenues and purchasing costs, then the profit per unit time of this solution is always less than the optimal average profit (i.e., the profit per unit time associated to the model of maximum profit).

2.2. Fundamental assumptions

In this paper we consider the following assumptions concerning the inventory control system:

(i) The replenishment rate is infinite.
(ii) The lead time is zero.
(iii) No shortage is allowed.
(iv) The unit purchasing cost and the selling price per unit are constant.
(v) The ordering cost per order is known and constant.
(vi) The demand rate is a function of the instantaneous stock level.
(vii) The holding cost per time-unit is a known function of the stock on display.

The assumptions (i)–(v) are frequently considered in the economic order quantity literature when the life cycle of the product has attained its maturity stage.

In addition, we assume the following functions for the inventory system considered in this paper:

$$D(t) = \lambda[I(t)]^{\beta} \quad \text{where } \lambda > 0 \text{ and } 0 \leqslant \beta < 1 \tag{1}$$

$$H(x) = hx^{\gamma} \quad \text{where } h > 0 \text{ and } \gamma \geqslant 1 \tag{2}$$

Expression (1) implies that the demand rate is an increasing function of the inventory level. That is, as the stock level decreases so does the demand rate. Thus, at the beginning of a cycle, the inventory level decreases rapidly because the demanded quantity is big at a higher level of stock. As more inventory is depleted, the rate of decrease of the stock level slows down. The inventory level elasticity of the demand rate, β, represents the relative change in demand rate with respect to the corresponding relative change