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An extension of inventory models with discretely variable holding costs

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ABSTRACT

In a recent paper, Alfares [2007. Inventory model with stock-level dependent demand rate and variable holding cost. *International Journal of Production Economics*, 108 (1–2), 259–265] presented an inventory model with a stock-dependent demand rate and variable holding costs. The analysis imposed a terminal condition that the inventory level at the end of the order cycle drop to zero and utilized a cost-minimization objective. However, with a stock-dependent demand rate, this approach does not provide an optimal profit. Allowing ending inventory to be nonzero, a profit-maximization model and solution methodology are developed. Computational results indicate a substantial improvement in the solution realized by the proposed approach.

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1. Introduction

It has long been acknowledged—and empirically established—that the demand rate of some retail items is an increasing function of the inventory level of the item. Baker and Urban (1988) presented a deterministic inventory model in which the demand rate is a function of the instantaneous inventory level; as the inventory level decreases during the order cycle, the displayed inventory and, consequentially, the demand rate decreases. Since then, a great deal of research has focused on stock-dependent demand models; see a recent review of the literature by Urban (2005).

Due to the endogenous nature of the demand rate, there are two distinguishing features in the analysis of stock-dependent demand models. First, the appropriate objective is profit maximization, not cost minimization. Revenues are fixed in traditional inventory models due to constant demand, but in a stock-dependent demand model, the demand rate is affected by the inventory decisions made. Thus, maximizing profits would encour-

age a higher demand rate (for profitable items) by maintaining higher inventory levels, whereas cost minimization would essentially attempt to reduce demand, which is counterintuitive to most retail enterprises. Second, it may be desirable to have a positive inventory at the end of the order cycle, even with deterministic demand. While a positive ending inventory will incur higher holding costs, it will also allow us to maintain a higher inventory level, realize a higher subsequent demand rate, and potentially receive greater profits.

A notable extension of stock-dependent demand models incorporates variable holding costs. Weiss (1982) noted that variable holding costs are appropriate when the value of an item decreases the longer it is in stock; Ferguson et al. (2007) recently indicated that this type of model is suitable for perishable items in which price markdowns or removal of aging product are necessary. Alfares (2007) states that more sophisticated storage facilities and services may be needed for perishable items if they are kept for longer periods of time. Goh (1994) first considered a stock-dependent demand model with variable holding costs, and assumed that the unit holding cost is a nonlinear continuous function of the time the item is in stock or a nonlinear continuous function of the inventory level. Giri and Chaudhuri (1998) extended this

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model to account for perishable products. Chang (2004) then amended the Giri and Chaudhuri model to utilize a profit-maximization objective and to allow for a positive inventory level at the end of the order cycle.

Alfares (2007) recently investigated the situation in which the variable holding costs are discrete in nature—a step function of the time in stock—with successively increasing costs. He considered two scenarios: (1) the holding cost of the last storage period of an order cycle is applied retroactively to all storage periods, so the same holding cost is applied to all units in the cycle, and (2) the holding cost of a storage period is applied incrementally to each period, so the holding cost for each storage period is applied only to the units held in that period. However, the proposed models and solution algorithms are derived using a cost-minimization objective and impose a terminal condition of zero inventory at the end of each order cycle.

Therefore, the purpose of this note is to generalize the Alfares (2007) model on stock-dependent demand models with variable holding costs by using a profit-maximization objective and by allowing, but not necessarily requiring, the ending inventory of a review cycle to take on a positive value. This may be particularly important with variable holding costs; since the unit holding costs are increasing over time, a positive ending inventory can help keep the order cycle shorter and the subsequent unit holding cost lower while maintaining a higher inventory level and demand rate. We will analyze the retroactive and incremental scenarios, providing a solution methodology and comparing the results to previously published results.

2. Model development

The model under consideration is the traditional economic order quantity (EOQ) model with the following exceptions: (i) the demand rate is an increasing function of the instantaneous inventory level, and (ii) the holding cost is an increasing step function over time. The notation of Alfares (2007) is used in the model development:

- Q order-up-to level
- T cycle length (order interval)
- t time from the beginning of a cycle ($0 \leq t \leq T$)
- q(t) instantaneous inventory level at time t; hence, $q(0) = Q$ is the inventory level at the beginning of an order cycle, and $q(T) \geq 0$ is the inventory level at the end of the cycle
- k replenishment (ordering) cost
- h_i holding cost per unit per unit time for storage period i (from t_{i-1} to t_i)

Since variable holding costs are frequently the result of aging product, we will distinguish between the sale of “fresh” items and that of “older” items. As consumers would tend not to purchase the older items (at the same price) if there are fresh items available, we will assume that the items remaining at the end of an order cycle will

be disposed of separately, perhaps sold through a secondary market or simply discarded. We define γ to be the gross profit per unit of fresh items (the selling price less the unit cost of goods sold) and define η to be the gross profit per unit of the items remaining at the end of the order cycle (the salvage value of the older items less the unit cost of goods sold less any disposal cost necessary). Ferguson et al. (2007) note that older product may be sold to restaurants or donated to food banks; however, it may be necessary to simply dispose of the product, so η may be negative if the salvage value is less than the purchase cost of the items.

The specific functional form of demand considered by Alfares (2007) is the multiplicative (power) function

$$R(q) = D[q(t)]^\beta \quad D > 0, 0 < \beta < 1 \tag{1}$$

The inventory function with respect to time can then be determined by evaluating the differential equation $dq/dt = -D[q(t)]^\beta$ with the initial condition of $q(0) = Q$:

$$q(t) = \begin{cases} (Q^{1-\beta} - [D(1-\beta)t])^{1/(1-\beta)} & \text{for } t \leq \frac{Q^{1-\beta}}{D(1-\beta)} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

We will analyze the two alternative formulations of the variable holding cost—retroactive and incremental—as presented by Alfares (2007).

2.1. Retroactive holding cost

In this situation, the unit holding cost is discrete in nature, and increases as the time in storage increases, $h_1 < h_2 < \dots < h_n$, for storage periods 1 through n, respectively. A retroactive holding cost implies that the holding cost of the last storage period is applied retroactively to all previous periods in the order cycle. That is, if the cycle length is t_1 or less, the unit holding cost is h_1 per time period; if the cycle length is between $t_1 < T \leq t_2$, all inventory (retroactively) is charged a holding cost of h_2 per unit per time period; etc. Since the same holding cost will be applied to all units in the cycle, we only need to determine the average inventory level for the entire order cycle:

$$\begin{aligned} \bar{q} &= \frac{1}{T} \int_0^T (Q^{1-\beta} - [D(1-\beta)t])^{1/(1-\beta)} dt \\ &= \frac{Q^{2-\beta} - [Q^{1-\beta} - D(1-\beta)T]^{2-\beta/(1-\beta)}}{D(2-\beta)T} \end{aligned} \tag{3}$$

The net profit per unit time, π , consists of the gross profit for the fresh and older product less the procurement cost and the holding cost:

$$\pi = \frac{\gamma[Q - q(T)] + \eta q(T)}{T} - \frac{k}{T} - h\bar{q} \tag{4}$$

where h is the corresponding value of $h = h_i$ for $t_{i-1} < T \leq t_i$. Note, the special case in which $\eta = 0$ is equivalent to the model in which the stock remaining at the time of replenishment is carried over to the next order cycle.

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