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Bootstrap confidence interval estimates of the bullwhip effect

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Abstract

This paper introduces the confidence interval estimate for measuring the bullwhip effect, which has been observed across most industries. Calculating a confidence interval usually needs the assumption about the underlying distribution. Bootstrapping is a non-parametric, but computer intensive, estimation method. In this paper, a simulation study on the behavior of the 95% bootstrap confidence interval for estimating bullwhip effect is made. Effects of sample size, autocorrelation coefficient of customer demand, lead time, and bootstrap methods on the 95% bootstrap confidence interval of bullwhip effect are presented and discussed.

Keywords: The bullwhip effect; Bootstrap; Autocorrelation; Supply chain

1. Introduction

The bullwhip phenomenon (or effect) referring to increase of demand variability further upstream in the supply chain has been observed or recognized in industry for a long time. The phenomenon can potentially cause instability in the supply chain and increase the cost of supplying goods to customer demand. The first recognition of this phenomenon can be traced back to Forrester [9]. Other earlier papers making a major contribution to understanding of bullwhip phenomenon include Blanchard [2], Blinder [3,4], Kahn [16], and Sterman [22]. Recently, Lee et al. [17,18] popularized the term ‘‘bullwhip effect’’, and analyzed four potential sources of the bullwhip effect: demand signal processing, rationing game, order batching, and price variations through simple mathematical models, which focuses on the retailer–supplier relationship and considers a first-order autoregressive (abbreviated as AR (1)) demand process.

Lee et al. [17,19] identified, moreover, the bullwhip effect as a natural consequence of demand signal processing, which refers to the situation where demand is non-stationary and one uses past demand information to update forecasts. Chen et al. [6–8] are early papers that link forecasting method with the bullwhip effect. Using

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an AR (1) demand process similar to Lee et al. [17], they quantify the magnitudes of the bullwhip effect resulting from moving averages, exponential smoothing, and other forecasting methods. Zhang [23] continued the study of Chen et al. and derived the bullwhip effect measure for the minimum mean-squared error forecasting method.

The Bullwhip effect is generally regarded as a performance index to respond to the instability in a serial supply chain. In practice, when applying the proposed measures to measuring the bullwhip effect, lead time and autocorrelation coefficient of demand process should be known. Lead time is the elapsed time between releasing an order and receiving it. In many literatures, lead time is regarded as a controllable decision variable and can be decomposed into several components, each having a crashing cost for the respective reduced lead time [21]. However, the exact autocorrelation coefficient of demand process is usually unknown because the number of observations collected from customer demand process is finite during a limited time horizon As a result, it is replaced by a sample autocorrelation coefficient, and this gives the measured bullwhip effect, a point estimate of the exact bullwhip effect.

In addition to the point estimator, interval estimation is important for the statistical inference on bullwhip effect of a particular supply chain. In this paper, we focus the investigation on the confidence interval of bullwhip effect. Calculation of the confidence interval for the bullwhip effect usually needs aware of the underlying distribution, but it could be difficult to know or obtain. Thus, we develop the confidence interval based on the bootstrap principle. Bootstrapping introduced by Efron [10,11,13] is a statistical method, which is non-parametric or free from assumptions of distribution.

The following section presents a simple replenishment model, which is the same model with that one of Lee et al. [17] considered in their literature. Taking into account the replenishment model into account, the bootstrap estimation technique and bootstrap confidence interval for the exact bullwhip effect are explained in Section 3. Although the problem we address is of practical interest, this paper is simulation based, which allows us to conduct controlled experiments. Thus, in Section 4 we describe the simulations that were run to evaluate the performance of bootstrap estimation for the exact bullwhip effect. Finally, we conduct a sensitivity study on the performance of this new measure and provide some concluding remarks.

2. A simple supply chain model

2.1. Replenishment policy

Assume a retailer–supplier system, where a single item and order-up-to S inventory policy are considered. To simplify the model, excess inventory can be returned without cost, and excess demand is backlogged.

The timing of events during a replenishment period is as follows: at the beginning of each period t, the retailer order a single item of quantity q_t from the supplier. There is a lead time of L periods between ordering and receiving the goods. After that, the goods ordered L periods ago arrived. Finally, demand is realized and the available stock is used to meet the demand. A serially correlated demand process is assumed to follow the AR (1) model, and it is similar to Lee et al. [17] and Hayakawa [24]

$$D_t = d + \rho D_{t-1} + \varepsilon_t \tag{1}$$

where D_t is the demand in period t, ρ and d are constants such that $d > 0$ and $-1 < \rho < 1$, and ε_t is normally distributed with zero mean and variance σ^2.

2.2. The exact bullwhip effect

Given the unit holding cost, unit shortage penalty cost, and the unit ordering cost, Lee et al. [17] formulated the cost minimum problem in order to optimize the retailer's order quantity q_t and the order-up-to level S_t, which is the amount in stock plus on order (including those in transit) after the decision q_t has been made. The optimal order-up-to level S_t^* resulting from the cost minimum problem was given by

$$S_t^* = d\sum_{k=1}^{L+1}\frac{1-\rho^k}{1-\rho} + \frac{\rho(1-\rho^{L+1})}{1-\rho}D_{t-1} + K\sigma\sqrt{\sum_{k=1}^{L+1}\sum_{i=1}^{k}\rho^{2(k-i)}} \tag{2}$$