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Measuring financial risks with copulas

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Abstract

This paper is concerned with the statistical modeling of the dependence structure of multivariate financial data using the concept of copulas. We select some special copulas and identify the type of dependency captured by each one. We fit copulas to daily returns and simulate from the fitted models. We compare the effect of the choice of copula on risk measures and assess the variability of one-step-ahead predictions of portfolio losses. We analyze extreme scenarios and fit extreme value copulas to the block maxima and minima from daily returns. The stress scenarios constructed are compared to those obtained using models from the extreme value theory. We illustrate the usefulness of the copula approach using two stock market indexes.

JEL classification: G10; C15
Keywords: Copulas; Dependence measures; Financial risks; Extreme value theory

1. Introduction

For a long time, the statistical modeling in finance had basically relied on simplified assumptions. For example, series of financial returns were taken as realizations of some time series model with Gaussian innovations. In the multivariate setting, the multivariate normal distribution was frequently assumed but seldom contested. However, the multivariate normal assumption restricts the type of association between the (normal) margins to be linear. This is a drastic restriction, and it is by no means reasonable to assume the linear association as the only type of dependency which could be observed among financial series.

Recently, the availability of computer softwares and the resulting sophistication allowed financial products to be based on several (combinations of) variables. Such

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complex products may possess also complex dependence structures—linear, nonlinear, and tail dependence—and this had raised questions and highlighted concerns about the appropriateness of the multivariate normal assumption and the use of the correlation coefficient as the canonical measure of association (Embrechts, McNeil, & Strauman, 1999; Harvey & Siddique, 1999).

These considerations have driven analysts to consider practical issues, such as:

(1) What are the appropriate multivariate distributions to model financial data?
(2) Which dependence measures should be computed to appropriately explain the several types of association observed in financial data?
(3) What are the effects of the assumed dependence structure on estimates of financial risk measures?

The problem of fitting multivariate distributions to financial data has been investigated by several authors, including Embrechts et al. (1999), Jobson and Korkie (1981), and Mendes and Leal (2002). Discussions related to the role of the linear correlation coefficient and other measures of association outside the class of elliptical distributions have been investigated by Embrechts, McNeil, et al. and Embrechts, Lindskog, and McNeil (2001), among others. How model choice and assumptions affect conditional and unconditional estimates of risk measures has been a topic of several works, including Embrechts, Resnick, and Samorodnitsky (1998), and McNeil and Frey (1998).

Modeling the dependence structure among variables using copulas is an approach recently rediscovered by a number of authors, including Bouyé, Durrleman, Nikeghbali, Riboulet, and Roncalli (2000), Frees and Valdez (1998), and Joe (1997), among others. The copula function is a multivariate distribution with uniform marginals (formal definitions in Section 2). For every multivariate distribution with continuous marginals, there is a unique copula representing their dependence structure (Nelsen, 1998). The clear representation and interpretation of the dependence structure of multivariate data (bivariate data in particular) can be more efficiently accomplished using the concept of copulas. This follows from the uncountable variety of possible combinations of copula types and marginal distributions that can be assumed.

Simulations play an important role in finance. They are used to replicate the efficient frontiers, to price options, to estimate joint risks, and so on. However, the resulting risk measures computed and the conclusions drawn from the simulations depend upon the assumed model and on the quality of the data-generating algorithms. Using the concept of copulas, it is relatively easy to construct and simulate from multivariate distributions based on almost any choice of marginals and any type of dependence structure.

In this paper, we model the dependence structure of multivariate financial data using the concept of copulas. We select some special copulas and identify the type of dependency captured by each one. We fit copulas to daily returns and simulate from the fitted models. We compare the effect of the choice of copula on risk measures and assess the variability of one-step-ahead predictions of portfolio losses. We analyze extreme scenarios and fit extreme value copulas to the block maxima and minima from daily returns. Our contribution is to show especially to practitioners in the area of finance that more reliable conclusions will certainly be drawn from their simulations if a