Economic Modelling 49 (2015) 331–343

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

# Arbitrage and leverage strategies in bubbles under synchronization risks and noise-trader risks

Senren Tan [a], Zhuo Jin [a,*], Fuke Wu [b]

[a] Centre for Actuarial Studies, Department of Economics, The University of Melbourne, VIC 3010, Australia
[b] School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan, China

ARTICLE INFO

*Article history:*
Accepted 22 May 2015
Available online 9 June 2015

*Keywords:*
Bubbles
Synchronization risks
Market timing
Noise-trader risk
Limits of arbitrage
Market efficiency
Behavioral finance
Leverage

ABSTRACT

This paper develops a model that explains the causation of persistent bubbles when arbitrageurs have noticed the overpricing. It has been claimed that a sufficient proportion of arbitrageurs needs to sell over-priced stocks to correct the mis-pricing. However, because each arbitrageur tends to choose his optimal time of entering or exiting the market, there exists a lack of coordination in selling out. Thus, the bubble will continue growing for a considerable period. Moreover, our work incorporates trend followers' impacts on stock prices into the analysis of the duration of bubbles. The derived equilibrium trading strategy suggests that each arbitrageur will wait for a longer period before selling out, compared with the optimal strategy obtained from previous models where only arbitrageurs' coordination risks are considered. This paper presents defects in the Efficient Market Hypothesis. Further, having shown that the duration of mis-pricing is increasing in arbitrageur's leverage ratio, the model provides rationales for regulations on individual use of leverage. This is compatible with the findings in previous literature that macro-prudential policy tools, such as limiting the use of leverages can be more effective than traditional monetary and fiscal policies in taming asset overpricing.



## 1. Introduction

Pricing bubbles do exist. Famous examples include the Dutch tulip mania in the 17th century, the South Sea bubble in the 18th century, the Internet bubble a decade ago and more recently, the housing bubbles before the Global Financial Crisis (GFC). Specifically, Roche (2001), Honohan and Leddin (2006) and Moons and Hellinckx (2015) provided detailed accounts addressing the housing bubbles in Ireland before the GFC. Moreover, with a more globalized economic and financial system nowadays, Crowe et al. (2013) argued that the bursting of the housing bubbles in the U.S. had resulted in the prolonged financial turmoil globally, from which many countries are still suffering. Stein (2011) argued that the housing price bubble in the U.S. is directly related to excessive leverages. On the other hand, standard neoclassical economic theory predicts no existence of bubbles. For example, Santos and Woodford (1997) presented a backward induction argument over a finite time period and an assumption that all agents in the market are rational. Also, according to Fama (1965), the Efficient Market Hypothesis (EMH) suggests the absence of price bubbles, although it assumes the existence of irrational behavioral traders in the market. The intuition is that arbitrageurs can make profits at the cost of behavioral traders' irrationality in the market. Therefore, behavioral traders will leave the market and the market will finally become rational.

The validity of EMH and limits of arbitrage have drawn increasing attention since the 1980s. Shiller (1981) and Campbell and Kyle (1993) explained that the difficulty in accurately estimating fundamental values would significantly restrict arbitrageurs' ability to correct the mis-pricing, referred to as the “fundamental risk” in arbitrage.

On the other hand, Black (1986) introduced the term “noise-trader” in financial markets, referring to those who do not trade in markets based on perfect information, or those who trade because “they like to trade”. DeLong et al. (1990a) further indicated that due to arbitrageurs' risk aversion over short horizons, the existence of noise traders might limit arbitrages even if there is no fundamental risk. This is referred to as the “noise-trader risk”. In particular, Shleifer and Vishny (1997) argued that if the stock price increases further after a trader sells short, he may face a temporary loss in his account. The margin requirement may force the arbitrageur to liquidate some of his holdings exactly when the mis-pricing is the largest. Also, Barberis et al. (2001), Liu and Longstaff (2004), Allen and Gorton (1993), DeLong et al. (1990b) and Brunnermeier (2001) considered noise-trader risk in delaying price corrections by arbitrageurs, therefore resulting in bubbles.

A third reason for limits of arbitrage considers arbitrageurs' coordination problem in trading. Deciding an optimal time to carry out arbitrage strategies is a form of a social dilemma, similar to Tucker (1980), Akerlof (1982) and Mckelvey and Palfrey (1992). Moreover, Kreps

* Corresponding author.
*E-mail addresses:* raymondtsr@gmail.com (S. Tan), zjin@unimelb.edu.au (Z. Jin), wufuke@hust.edu.cn (F. Wu).

http://dx.doi.org/10.1016/j.econmod.2015.05.009
0264-9993/

et al. (1982) showed that information asymmetries among different agents in the market would yield cooperation in equilibrium. Also, Abreu and Brunnermeier (2003) argued that the robustness of both neoclassical and EMH arguments depends significantly on the assumption that all arbitrageurs notice the mis-pricing at a similar time. However, this assumption is unrealistic. Thus, it seems feasible to assume that arbitrageurs sequentially notice the mis-pricing.

Abreu and Brunnermeier (2002) argued that due to the sequential awareness of bubbles, arbitrageurs have a synchronization problem trying to sell mis-priced stocks. The mis-pricing will be corrected only if arbitrageurs' aggregate selling pressure exceeds boundedly rational noise-traders' demand. This is similar to models of currency attacks in Obstfeld (1996) and Morris and Shin (1998). Further, because of arbitrageurs' heterogeneous beliefs on fundamental value of a stock, each arbitrageur tends to "ride the bubble", instead of correcting the price immediately after he notices the mis-pricing. Therefore, Shiller (2000) claimed that the absence of bubbles as claimed in the neoclassical economic theory and the EMH is undermined. It should be noted that the introduction of arbitrageurs' competition in selling out before crashes occur does not appear in traditional currency attack literatures. Because of the fundamental risk, the noise-trader risk and the synchronization risk, a more realistic model considering these risks in arbitrage and justifying the causation of bubbles is needed.

This paper provides a model that measures the interaction of the noise-trader risk and arbitrageurs' synchronization risk in forming bubbles. Results in our model show that arbitrageurs will delay their trading for a substantial time period, even though they have noticed the existence of bubbles and are allowed to short sell their assets. Specifically, our model suggests that highly-levered arbitrageurs are more reluctant to sell their stocks short due to concerns of noise-trader risks, thereby further delaying the price correction.

Firstly, our model assumes two groups of players in the market, rational arbitrageurs and noise-traders. Rational arbitrageurs notice pricing bubbles and prepare to short sell shares, whereas noise-traders are trend followers unaware of the mis-pricing. Moreover, our model supposes that arbitrageurs sequentially notice the bubble and have dispersed views on the starting time of the mis-pricing. In addition, price correction is assumed to occur only when arbitrageurs' aggregate selling pressure exceeds noise-traders' demand. This coordination requirement to correct the bubble, combined with arbitrageurs' sequential awareness of the bubble, causes a delayed correction of mis-pricing. The above set-up relates to Burnside et al. (2011), where the authors argued that agents change their expectations according to the dynamics of positive and negative signals, therefore causing the boom–bust cycle.

However, previous literatures only accounted for arbitrageurs' coordination risks in determining optimal strategy in price correction. On the other hand, our model provides a quantitative explanation of noise-traders' risk as well as arbitrageurs' coordination risks on the optimal strategy in price correction. Our model, in addition to the direct trading cost, introduces an indirect cost measuring noise-traders' feedback effect on arbitrageurs' expected pay-off. One result in our model is that highly-levered arbitrageurs take longer time to short sell stocks after becoming aware of the mis-pricing. Therefore, a restriction on the maximal possible leverage usage would reduce arbitrageurs' waiting period. That is, the correction of mis-pricing will be earlier. This is compatible to the findings in Stein (2009).

Furthermore, Rubinstein (1989) argued that mutual knowledge and common knowledge would lead to different optimal strategies in the auction-like, electronic-mail game. In our model, it is possible that all arbitrageurs notice the mis-pricing and all arbitrageurs know that all arbitrageurs notice the mis-pricing. However, that each arbitrageur notices the bubble is mutual knowledge instead of common knowledge, due to the assumption of sequential awareness of bubbles. Each arbitrageur does not know for sure his order in noticing the bubble and infers optimal trading strategy based on his own information. Even if all arbitrageurs in fact notice the bubble, an individual arbitrageur does not know for sure that all other arbitrageurs notice the mis-pricing. It should be noted that both neoclassical and EMH approaches imply that the information that all arbitrageurs notice the bubble is common knowledge. Thus, our model derives a different equilibrium optimal strategy.

The remainder of this paper will be organized as follows. Section 2 demonstrates the model and indicates key elements of the model. Section 3 proves that we can restrict our analysis to trigger-strategies in equilibrium. Section 4 deals with individual equilibrium trading strategies under different situations and hence presents that bubbles will burst for different reasons. A numerical example is presented in Section 5 to illustrate the results. Additional remarks are provided in Section 6.

## 2. Formulation

This paper will discuss the case of an over-estimate in fundamental values that causes pricing bubbles. The under-pricing case shares a similar reasoning. There is a single risky asset with spot price process denoted by $p(t)$, while its fundamental value is denoted by $v(t)$. Growth rates of the spot and fundamental values are $g$ and $r$ respectively, with $g > r$ in the case of an optimistic estimate of fundamental improvements and $g < r$ when there is a pessimistic estimate. Prior to time 0 ($t = 0$), it is assumed that the spot value equals the fundamental value. Without loss of generality, we consider the time period $[0, \infty)$ and unitize the security's price to 1 at $t = 0$. From $t = 0$ onwards, under the constant growth rate assumption, $p(t) = e^{gt}$ and $v(t) = e^{rt}$.

In our model, it is assumed that between time 0 and some random time $t_0$, the increase in the stock price matches the fundamental improvement. That is, $p(t) = v(t) = e^{gt}$. We also assume that $t_0$ has an exponential prior distribution on $[0, \infty)$ with parameter $\lambda$. Hence, its distribution function is $F(t_0) = 1 - e^{-\lambda t_0}$. From $t_0$ onwards, only a proportion $1 - \beta(\cdot)$ of the spot price can be explained by fundamental movements. Here, $\beta(\cdot)$ represents the temporary mis-pricing, which is assumed to be continuous and increasing in time elapsed from $t_0$ onwards, $t - t_0$. For simplicity, we assume that the evolution of the fundamental value is continuous and has a growth rate of $r$ in perpetuity. That is, $v(t) = e^{gt_0 + r(t - t_0)}, \forall\ t > t_0$ and function $\beta(t - t_0) = 1 - e^{-(g-r)(t-t_0)}$. This paper will focus on the bubble case, that is, when $g > r$.

We assume that there are two types of market participants: rational arbitrageurs, such as hedge funds, and noise traders such as households, trend followers. Without loss of generality, we normalize the mass of arbitrageurs to 1. From $t_0$ onwards, it is assumed that rational arbitrageurs sequentially notice the mis-pricing. Specifically, we assume that traders notice the bubble uniformly between $t_0$ and $t_0 + \eta$, where $\eta$ is an exogenous parameter indicating the duration of arbitrageurs' awareness of the bubble. Furthermore, we assume that the bubble will finally burst exogenously at time $t_0 + \overline{\tau}$, even if arbitrageurs do not burst it endogenously before. Here, $\overline{\tau} > \eta$ is an exogenous parameter to be calibrated.

We define the set, $\mathcal{M} = \{0, 1, \cdots, N \mid t_N = t_0 + \eta, N \in \mathbb{Z}^+\}$, where $N$ represents the total number of arbitrageurs in the market. Then for $i \in \mathcal{M}$, an arbitrageur becoming aware of the bubble at $t_i$ believes that the posterior distribution of $t_0$ has a support of $[t_i - \eta, t_i]$. If $t_0 = t_i$, the arbitrageur is in fact the first to notice the mis-pricing. Whereas if $t_0 = t_i - \eta$, the arbitrageur is the last to observe any bubble. Hence, arbitrageur $t_i$'s posterior belief on the distribution function of $t_0$ is

$$\Phi(t_0 | t_i) = \frac{1 - e^{-\lambda(t_0 - (t_i - \eta))}}{1 - e^{-\lambda\eta}} = \frac{e^{\lambda\eta} - e^{\lambda(t_i - t_0)}}{e^{\lambda\eta} - 1}, \tag{2.1}$$

where $t_0 \in [t_i - \eta, t_i]$. However, it should be noted that since $t_0$ is random, the individual arbitrageur does not exactly know $t_0$ or his order in noticing the bubble. On the other hand, noise-traders assume the fundamental value to continue growing at a rate $g$ even after $t_0$. Under this assumption, unlike rational arbitrageurs, noise-traders still tend to buy more securities at higher prices. Noise-traders' excessive demand will absorb rational