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The optimum output quantity of a duopoly market under a fuzzy decision environment

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Abstract

The main purpose of this paper is to develop a new optimum output quantity decision analysis of a duopoly market under a fuzzy decision environment. To efficiently handle the fuzziness of the decision variables, the linguistic values, subjectively represented by the trapezoidal fuzzy numbers, are used to act as the evaluation tool of decision variables such as fixed cost and unit variable cost. This paper will apply fuzzy set theory to construct an optimum output quantity decision model based on aiming for the maximum profit of a duopoly market. By using this decision model, the decision-makers' fuzzy assessments with various variables can be considered in the decision process to assure more convincing and accurate decision-making.

Keywords: Trapezoidal fuzzy number; Cournot model; Stackelberg model; Optimum output quantity; Duopoly market

1. Introduction

To maximize profit and minimize overall cost, the optimum output quantity decision has been an important issue for industrial organization. In a duopoly market, two models can be used to optimize the output quantity. One of them is the Cournot model [1–3], and the other is the Stackelberg model [4]. The Cournot and Stackelberg models are similar because, in both, competition occurs in terms of quantity. The Cournot model forms a situation in which each firm chooses its output independently. The Stackelberg model is a two-stage leadership in which the leader chooses its output quantity before the follower does. The follower then notes the leader's choice of output quantity and chooses its own output quantity [5]. Both Cournot and Stackelberg model play a vital role in such fields as economics, behavioral sciences, management, and politics [6–8].

Alepuz and Urbano [9] analyzed how learning behavior can change the outcome of competition in a duopoly industry facing demand uncertainty. They found that each firm will increase its first period quantity for the myopic choice to make price a more informative signal. Wang [10] studied and compared fee and royalty licensing in a differentiated Cournot duopoly. The study results showed licensing by a royalty may be better than a fixed-fee from the

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viewpoint of the patent-holding firm. However, the consumer preferred the fixed-fee licensing. Barr and Saraceno [11] examined the effects of both environmental and organizational factors on the outcome of repeated Cournot games.

Eiselt [12] examined a facility planner's advantage resulting from knowledge of his competitor's opinion in the location Stackelberg games. Some valuable findings are obtained. For example, given perfect information, the leader always has an advantage over the follower. Lavigne et al. [13] presented a general method to study the electricity market of a country or region, under various pricing mechanisms. The study results showed monopolistic pricing chosen by the producer to minimize its costs while knowing the optimal consumers' reaction to the proposed price of electricity leads to a Stackelberg-type equilibrium. Nie [6] explored discrete time dynamic Stackelberg games with an open loop complete state. The study pointed out that both feedback and closed loop dynamic Stackelberg games with complete information are valuable in explaining some social and economic phenomena. Yang and Zhou [8] analyzed the effects of the duopolistic retailers' different competitive behaviors - Cournot Collusion and Stackelberg - on the optimal decisions of the manufacturer and the duopolistic retailers themselves. The results showed that the total profit of the duopolistic retailers who act as the followers will exceed the more powerful manufacturer's profit as long as the dissimilarity between the duopolistic retailers' market demands is sufficient.

In a duopoly market, four patterns to market structure can be formed. They are: (1) both companies A and B are followers; (2) company A is a leader and company B is a follower; (3) company B is a leader and company A is a follower; (4) both companies A and B are leaders. In pattern (2) and (3), the leader makes the optimum output quantity decision by considering the response function of the follower, then, the follower decides his optimum output quantity based on the leader's decision. In pattern (1), the optimum output quantities of companies A and B can be solved by the simultaneous-equation models constructed by their response function respectively. In pattern (4), the optimum output quantities of companies A and B can be achieved when they are recognized as leaders.

In conventional precision-based models of duopoly, decision variables are expressed in crisp values [14,15]. However, because of the insufficiency and uncertainty of information in the decision environment, it is difficult to find the exact economic assessment data, such as the prices of products, volume of activity, per unit variable costs, and total fixed costs. Therefore, the precision-based decision may be ineffective. In fact, when decision-makers decide, they assess based on their professional knowledge, experience, and subjective judgment. Linguistic values, such as "about 2000 dollars", "about 40%", are usually used to suggest their estimations. Fuzzy set theory can play a significant role in this decision-making environment.

Fuzzy set theory was introduced by Zadeh [16] to solve problems in which a source of vagueness existed. Linguistic values can be expressed fittingly by the approximate reasoning of fuzzy set theory [17]. To deal with the ambiguities involved in the process of linguistic estimations effectively, the trapezoidal fuzzy numbers are used to characterize fuzzy measure of linguistic values [18]. At the same time, combining the Cournot and Stackelberg models, a fuzzy duopoly model which considers the factors of market demand, business cost and market position is developed to answer the optimum output quantity of duopoly market under a fuzzy decision environment.

This paper is organized as follows. Section 2 introduces fuzzy set theory. The fuzzy model of duopoly is developed in Section 3. A numerical example to explain the computational process of a fuzzy model of duopoly will be presented in Section 4. Finally, conclusions are given in Section 5.

2. Fuzzy set theory

In this section, some concepts of fuzzy set theory used in this paper are briefly introduced.

2.1. Trapezoidal fuzzy numbers

A fuzzy number A [19] is described as a special fuzzy subset of real numbers whose membership function f_A satisfies five conditions. (1) f_A is a continuous mapping from $\Re$ (real line) to a closed interval [0, 1], (2) $f_A(x) = 0$, for all $x \in (-\infty, c] \cup [d, \infty)$, (3) $f_A(x)$ is strictly increasing in the interval $[c, a]$, (4) $f_A(x) = 1$, for all $x \in [a, b]$, (5) $f_A(x)$ is strictly decreasing in the interval $[b, d]$. Where c, a, b, d are real numbers, and $-\infty < c \le a \le b \le d < \infty$. For convenience, f_A^L is named as the left membership function of fuzzy number A, defining $f_A^L(x) = f_A(x)$, for all $x \in [c, a]$; and f_A^R is named as the right membership function of fuzzy number A, defining $f_A^R(x) = f_A(x)$, for $x \in [b, d]$. The fuzzy number A in $\Re$ is a trapezoidal fuzzy number if its membership function $f_A : \Re \to [0, 1]$ is