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Dynamics analysis and chaos control of a duopoly game with heterogeneous players and output limiter

Jian-guo Du [a,b], Yue-qian Fan [a,*], Zhao-han Sheng [c], Yun-zhang Hou [d]

[a] School of Management, Jiangsu University, Zhenjiang 212013, China
[b] Computational Experiment Center for Social Science at Jiangsu University, Zhenjiang 212013, China
[c] School of Engineering and Management, Nanjing University, Nanjing 210093, China
[d] Shanghai Logistic Institute, Fudan University, Shanghai 200433, China

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ABSTRACT

In actual economies, players sometimes would offer an upper limiter to their output due to capacity constraints, financial constraints and cautious response to uncertainty in the world, or offer a lower limiter to their output due to economies of scale or break-even consideration. In this paper, we discuss a dynamic duopoly game with heterogeneous players by assuming that one of them imposes an upper limiter on output, and the other one imposes a lower limiter. We analyze how the limiter affects the dynamics of output and the performance of players, and explore the number of the equilibrium points and the distribution of conditioned equilibrium points of the model. We then discuss the stable region of conditioned equilibrium. The theoretical results and numerical experiments show that adding appropriate limiter can make the system more robust, and even get rid of its chaos. The numerical results show that chaotic output dynamics can be beneficial to one firm but harmful to the other, and can also be harmful to both of them, and also show that adding appropriate limiter can improve the performance of the player.

1. Introduction

Since Cournot (1838) introduced the first well-known model, which gives a mathematical description of competition in a duopolistic market consisting of two quantity-setting firms who produce homogeneous goods and make the optimal output choice through assuming the last values taken by competitors in every step, there have been many researches based on it (see, e.g., Agiza and Elsadany, 2003, 2004; Agiza et al., 2001, 2002; Ahmed and Agiza, 1998; Ahmed et al., 2000; Bischi and Naimzada, 1999; Ding et al., 2009; Dubiel-Teleszynski, 2011; Fan et al., 2012; Fanti and Gori, 2012a,b; Hassan, 2004; Kopel, 1996; Onozaki et al., 2003; Puu, 1991, 1996; Rand, 1978; Tramontana, 2010; Yassen and Agiza, 2003; Zhang et al., 2007). All these researches, based on different assumptions which involve demand function (see Agiza et al., 2001, 2002), cost function (see Agiza et al., 2002; Yassen and Agiza, 2003), and the players' expectations about the decisions of competitors (see Agiza and Elsadany, 2003, 2004; Bischi et al., 2010), have shown that the Cournot adjustment process may not converge to a Nash equilibrium, and may lead to periodic cycles and deterministic chaos. Some researches assume that the players have homogeneous expectations (see Agiza et al., 2001, 2002; Ahmed and Agiza, 1998, 2000; Bischi and Naimzada, 1999; Hassan, 2004; Yassen and Agiza, 2003), while others consider that the players adopt different decision mechanism as regards expectation formation (see, e.g., Agiza and Elsadany, 2003, 2004; Ding et al., 2009; Dubiel-Teleszynski, 2011; Fan et al., 2012; Fanti and Gori, 2012a,b; Onozaki et al., 2003; Tramontana, 2010; Zhang et al., 2007). In particular, Fanti and Gori (2012a) consider differentiated products, and make progress in dynamic role played by the degree of product differentiation.

It should be noted that, almost all the current literature implies that outputs are not constrained, so the evolution orbit of attractors (especially the strange one) in these duopoly models can stretch out and fold freely. However, in actual economies, it is commonly observed that competitive firms would respond to the fluctuating prices cautiously by limiting their production, which may restrict the variation of attractors in economic systems, and make the system's dynamics different from that in the existing literature. Thus, the present paper studies the impact of this limitation on dynamics of duopoly game model. We consider an output duopoly game with two heterogeneous players by assuming that one of them imposes an upper limiter on output, and the other one imposes a lower limiter, and we focus on the dynamic role played by

* Corresponding author. Tel.: +86 13913853939; fax: +86 511 88780186.
E-mail addresses: jgdu2005@163.com (J. Du), yqfan_jduniv@163.com (Y. Fan).

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these limiters. Imposing limiters on state variable can suppress the chaos in systems, which was analytically and numerically explored by Du et al. (2010), He and Westerhoff (2005), Wagner and Stoop (2000), Stoop and Wagner (2003), and was termed as the limiter method by He and Westerhoff (2005). Zhang and Shen (2001) identify that imposing limiters on state variable can control the chaos by compressing the evolution orbit of the chaotic attractor. Unfortunately, there are no theoretical results to assure the fact that chaos can be suppressed by adding a simple limiter. Moreover, the researches mentioned above have not discussed the impact of limiter on the equilibrium of economic system and on the performance of the player.

To fill this gap, we focus on unraveling stabilizing mechanism of limiter method to reduce the fluctuation. We also discuss whether chaotic dynamics in duopoly game model are desirable or not by using a method similar to that in Matsumoto (2003). However, in contrast with Matsumoto's research, which considers price instability, and measures the variation of the utility of two consumers by using long-run average utility, we study output instability, and introduce aggregate profits to measure the performance changes of two producers. The results in this paper cannot support Matsumoto's result that chaotic fluctuations can be preferable to a stationary state.

The remainder of this paper is organized as follows. Section 2 introduces an output duopoly game with bounded rationality, and examines the dynamics of the model. Section 3 turns to discuss the output duopoly game having an upper limiter and a lower limiter, and analyzes the impact of output limiters on dynamics. The numerical simulation of dynamics and chaos control, and a comparison of the player's performance before the control and after control are given in Section 4. The final section concludes the paper.

2. The output model without limiter and its dynamics

2.1. The model

The output model we introduce here is based on the assumption that the two firms (players) produce a homogenous product. The generic firm is indexed with $i = \{1, 2\}$. The strategy space is the choice of the output, and the decision-making takes place in the discrete time periods $t = 0, 1, 2, \ldots$. $q_i(t)$ represents the output of firm i at time t. As usual in duopoly models, the price p of the goods at time t is determined by the total supply $Q(t) = q_1(t) + q_2(t)$ through an inverse demand function:

$$p = f(Q) = a - bQ, \tag{1}$$

where a and b are positive constants, and a is the highest price in the market. We assume that the cost function has the nonlinear form:

$$C_i(q_i) = c_i + d_i q_i + e_i q_i^2, i = 1, 2, \tag{2}$$

where the positive parameter c_i is the fixed cost of firm i. As a rule, the marginal cost of firm $i (i = \{1, 2\})$ must be less than the highest price of the goods in the market. Therefore, $d_i + 2e_i q_i < a$, $i = 1,2$. Hence the single profit of firm i at time t is:

$$\Pi_i(q_1, q_2) = q_i(t)(a - bQ(t)) - \left(c + d_i q_i(t) + e_i q_i^2(t)\right), i = 1, 2. \tag{3}$$

The marginal profit of firm i at time t is:

$$\frac{\partial \Pi_i(q_1, q_2)}{\partial q_i} = a - bQ(t) - bq_i(t) - d_i - 2e_i q_i(t), i = 1, 2. \tag{4}$$

We assume that the two players do not have a complete knowledge of the market. In games, players behave adaptively, following a bounded rationality adjustment process based on a local estimate of the marginal profit $\partial \Pi_i / \partial q_i$ (see Agiza et al., 2002; Ahmed and Agiza, 1998; Bischi and Naimzada, 1999; Hassan, 2004; Yassen and Agiza, 2003). That is to say, if a firm thinks the marginal profit at time t is positive, it will increase its production at time $t + 1$; whereas if the marginal profit is negative, it will decrease its production. Then the dynamic adjustment mechanism of the firm i can be modeled as:

$$q_i(t+1) = q_i(t) + \alpha_i q_i(t) \frac{\partial \Pi_i(q_1, q_2)}{\partial q_i}, i = 1, 2, \tag{5}$$

where α_i is positive parameter representing the speed of adjustment. Taking Eq. (4) into Eq. (5), we can obtain the duopoly model with bounded rational players as the following:

$$q_i(t+1) = q_i(t) + \alpha_i q_i(t)[a - bQ(t) - (b + 2e_i)q_i(t) - d_i], i = 1, 2. \tag{6}$$

2.2. Equilibrium points and local stability

In order to make the solution of the output duopoly model economically significant, we study the nonnegative stable state solution of the model in this paper. The equilibrium solution of the dynamics system (6) is the following algebraic nonnegative solution:

$$\begin{cases} q_1(a - bQ - (b + 2e_1)q_1 - d_1) = 0 \\ q_2(a - bQ - (b + 2e_2)q_2 - d_2) = 0 \end{cases}. \tag{7}$$

From Eq. (7), we can get four fixed points:

$$E_0 = (0, 0), E_1 = \left(\frac{a - d_1}{2b + 2e_1}, 0\right), E_2 = \left(0, \frac{a - d_2}{2b + 2e_2}\right), E^* = (q_1^*, q_2^*),$$

where $q_1^* = \frac{(a-d_1)(2b+2e_2) - b(a-d_2)}{3b^2 + 4be_1 + 4be_2 + 4e_1e_2}$, $q_2^* = \frac{(a-d_2)(2b+2e_1) - b(a-d_1)}{3b^2 + 4be_1 + 4be_2 + 4e_1e_2}$.

Since E_0, E_1 and E_2 are on the boundary of the decision set $D_1 = \{(q_1, q_2) | q_1 \geq 0,\ q_2 \geq 0\}$, they are defined as boundary equilibrium. E^* is the unique Nash equilibrium provided that

$$\begin{cases} (a - d_1)(b + 2e_2) - b(d_1 - d_2) > 0 \\ (a - d_2)(b + 2e_1) - b(d_2 - d_1) > 0 \end{cases}. \tag{8}$$

The Nash equilibrium E^* is located at the intersection of the two reaction curves which represent the locus of points of vanishing marginal profits in Eq. (4). In the following, we assume that Eq. (8) is satisfied, so the Nash equilibrium E^* exists.

The study of the local stability of equilibrium points is based on the eigenvalues of the Jacobian matrix of the Eq. (6)

$$J = \begin{bmatrix} 1 + \alpha_1(a - (4b + 4e_1)q_1 - bq_2 - d_1) & -\alpha_1 bq_1 \\ -\alpha_2 bq_2 & 1 + \alpha_2(a - bq_1 - (4b + 4e_2)q_2 - d_2) \end{bmatrix}. \tag{9}$$

As regards the conditions for the fixed point to be stable (see Agiza et al., 2001, 2002), we have the following result.

Proposition 1. *The boundary equilibrium points E_0, E_1 and E_2 are unstable.*

Proof. See in Appendix A. ■

In order to study the local stability of Nash equilibrium $E^* = (q_1^*, q_2^*)$, we estimate the Jacobian matrix at E^*, which is:

$$J(E^*) = \begin{bmatrix} 1 - \alpha_1 f_1 & -\alpha_1 bq_1^* \\ -\alpha_2 bq_2^* & 1 - \alpha_2 f_2 \end{bmatrix}, \tag{10}$$

where $f_1 = 2(b + e_1)q_1^*$ is positive, $f_2 = 2(b + e_2)q_2^*$ is positive.

The characteristic equation of Eq. (10) is:

$$P(\lambda) = \lambda^2 - \mathrm{Tr}\,\lambda + \mathrm{Det} = 0,$$