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Dynamic feedback Stackelberg games with non-unique solutions☆

Pu-yan Nie[a,*], Ming-yong Lai[b], Shu-jin Zhu[b]

[a] *Institute of Industrial Economics, Jinan University, Guangzhou, 510632, PR China*
[b] *College of Economics and Trade, Hunan University, Changsha, 410079, PR China*

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Abstract

Stackelberg games, which was originally introduced by Stackelberg, are widely applied in such fields as economics, management, politics and behavioral sciences. Stackelberg games can be modelled as a bi-level optimization problem. There exists an extensive literature about static bi-level optimization problems. However, the studies on dynamic bi-level optimization problems are fairly scarce in spite of the importance in explaining and predicting some phenomena rationally. In this paper, we consider discrete time dynamic Stackelberg games with feedback information. In general, the lower-level strategies are non-unique in practice. For a unique solution, dynamic programming algorithms have been presented with multiple players. We revisit dynamic programming for feedback information dynamic Stackelberg games with non-unique lower-level solution. First, we define some kind of solutions related to the decisions styles. Then, we analyze them, respectively. Moreover, dynamic programming algorithm is successful in solving solve feedback information dynamic Stackelberg games with non-unique lower-level solutions.

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Keywords: Discrete time dynamic models; Stackelberg games; Dynamic programming algorithm; Lower-level problems

1. Introduction

In many decision processes there exists a hierarchy of decision makers, which can be modelled by bi-level programming problems and was introduced by Von Stackelberg [12]. There exist extensive researches on bi-level optimization [11]. Under certain conditions, a bi-level optimization problem can be reformulated as a mathematical program with equilibrium constraints (MPEC) [6], which has recently drawn much attention in the optimization community [7,9,10]. However, the study on dynamic bi-level optimization is relatively scarce. Dynamic bi-level optimization was first introduced by Chen and Cruz [4] and subsequently studied by a number of authors [2,3,5].

The discrete time dynamic optimization problem has comprehensive applications in economics and management sciences and has been studied extensively [3]. In this paper, we consider a dynamic Stackelberg game that may be

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* Corresponding author.
E-mail address: pynie2005@yahoo.com.cn (P.-y. Nie).

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modelled as a discrete time dynamic bi-level optimization problem, where the upper-level state variables are influenced by the decisions of the leader, and the lower-level state variables are also related to the decisions of leader and the followers. The model was introduced in [8], which is listed as follows.

Let us give the formal statement of the problem. The discrete time periods are denoted $t = 0, 1, \dots, T$, and N is the number of the followers in the game.

The variables involved in the problem are listed as follows:

Vectors $x_t \in \mathcal{X} \subset R^{m_0}$ denote the state of the leader at time $t = 0, 1, \dots, T$.

Vectors $y_t^\nu \in \mathcal{Y}^\nu \subset R^{m_\nu}$ denote the state of the νth follower at time $t = 0, 1, \dots, T$. The followers' state variables at time t are collectively denoted $y_t = (y_t^1, y_t^2, \dots, y_t^N) \in \mathcal{Y} \subset R^m$ with $m = m_1 + m_2 + \cdots + m_N$.

Vectors $u_t \in \mathcal{U} \in R^{n_0}$ denote the decision variables for the leader at time $t = 0, 1, \dots, T - 1$.

Vectors $v_t^\nu \in \Pi_t^\nu(x_t, y_t^\nu, u_t) \subset \mathcal{V}^\nu \subset R^{n_\nu}$ denote the decision variables for the νth follower at time $t = 0, 1, \dots, T - 1$. (The definition of $\Pi_t^\nu(x_t, y_t^\nu, u_t)$ is given below). The followers' decision variables at time t are collectively denoted $v_t = (v_t^1, v_t^2, \dots, v_t^N) \in \mathcal{V} \subset R^n$ with $n = n_1 + n_2 + \cdots + n_N$.

Moreover, we denote

$$v^\nu := (v_0^\nu, v_1^\nu, \dots, v_{T-1}^\nu), \quad v := (v^1, v^2, \dots, v^N), \quad y^\nu := (y_0^\nu, y_1^\nu, \dots y_{T-1}^\nu), \quad y := (y^1, y^2, \dots, y^N).$$

The state variables $\{x_t\}_{t=0}^T$ and $\{y_t\}_{t=0}^T$ are governed by the systems of state transition equations

$$x_{t+1} = F_t(x_t, u_t), \quad t = 0, 1, \dots, T - 1, \tag{1.1}$$

$$y_{t+1}^\nu = f_t^\nu(x_t, y_t^\nu, u_t, v_t^\nu), \quad t = 0, 1, \dots, T - 1, \nu = 1, 2, \dots, N \tag{1.2}$$

with $x_0 = a_0, y_0^\nu = b_0^\nu$ being given as the initial states of the leader and the νth followers, $\nu = 1, 2, \dots, N$, respectively.

Let the set of admissible decisions of the leader be given by

$$\Pi^0(x) := \{u | h_t^0(x_t, u_t) \le 0, t = 0, 1, \dots, T - 1\},$$

and the set of admissible decisions of the followers $\nu = 1, 2, \dots, N$ be given by

$$\Pi^\nu(x, y^\nu, u) := \{v^\nu | h_t^\nu(x_t, y_t^\nu, u_t, v_t^\nu) \le 0, t = 0, 1, \dots, T - 1\},$$

where h_t^0 and h_t^ν, $\nu = 1, 2, \dots, N$, are some given functions.

For convenience, we denote

$$\Pi_t^0(x_t) := \{u_t | h_t^0(x_t, u_t) \le 0\},$$
$$\Pi_t^\nu(x_t, y_t^\nu, u_t) := \{v_t^\nu | h_t^\nu(x_t, y_t^\nu, u_t, v_t^\nu) \le 0\},$$
$$\Pi(x, y, u) := \Pi^1(x, y^1, u) \times \Pi^2(x, y^2, u) \times \cdots \times \Pi^N(x, y^N, u),$$
$$\Pi_t(x, y, u) := \Pi_t^1(x, y^1, u) \times \Pi_t^2(x, y^2, u) \times \cdots \times \Pi_t^N(x, y^N, u).$$

Note that these sets depend only on the decision u of the leader, since the state variables x_t and y_t^ν are completely determined by the state transition Eqs. (1.1) and (1.2), respectively.

Given the initial states $(x_0, y_0) \in \mathcal{X} \times \mathcal{Y}$, the problem is formally stated as follows:

$$\begin{aligned} &\text{minimize} \quad J^0(x_0, y_0, u, v) \\ &\text{subject to} \quad V^\nu(x_0, y_0, u) = J^\nu(x_0, y_0, u, v^\nu) \\ &\qquad\qquad u \in \Pi^0(x), \quad v^\nu \in \Pi^\nu(x, y^\nu, u), \quad \nu = 1, 2, \dots, N, \end{aligned} \tag{1.3}$$

where

$$J^0(x_0, y_0, u, v) := G_T(x_T, y_T) + \sum_{t=0}^{T-1} G_t(x_t, y_t, u_t, v_t),$$

$$J^\nu(x_0, y_0, u, v^\nu) := g_T^\nu(x_T, y_T^\nu) + \sum_{t=0}^{T-1} g_t^\nu(x_t, y_t^\nu, u_t, v_t^\nu),$$

and $V^\nu(x_0, y_0, u)$ are the optimal value functions for followers $\nu = 1, 2, \dots, N$ defined by

$$V^\nu(x_0, y_0, u) := \min\{J^\nu(x_0, y_0, u, v^\nu) | (1.1), (1.2) \text{ and } v^\nu \in \Pi^\nu(x, y^\nu, u)\}.$$